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CALCULATING LAMINAR AND TURBULENT BOUNDARY
LAYERS FOR TWO-DIMENSIONAL TIME-DEPENDENT
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A Computer Program for Calculating Laminar and Turbulent Boundary Layers for Two-Dimensional Time-Dependent Flows

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TWO-DIMENSIONAL TIME-DEPENDENT FLOWS**

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by

Tuncer Cebeci* and Lawrence W. Carr

SUMMARY

There are many unsteady boundary-layer problems in which it is necessary to account for fluctuations in the external flow. These fluctuations may change in direction and magnitude and, in most turbulent flow cases, may be regarded as low frequency fluctuations superimposed on the turbulence energy spectrum. The computer program described here provides solutions of two-dimensional equations appropriate to laminar and turbulent boundary layers for boundary conditions with an external flow which fluctuates in magnitude; it can readily be extended to the more general case. It is based on the numerical solution of the governing boundary-layer equations by an efficient two-point finite-difference method developed by Keller and Cebeci [1]. An eddy-viscosity formulation is used to model the Reynolds shear-stress term. Here, after briefly describing the main features of the method, we provide instructions for the computer program with a listing, and present sample calculations to demonstrate its usage and capabilities for laminar and turbulent unsteady boundary layers with an external flow which fluctuates in magnitude. For further details of the method, the reader is referred to reference 2.

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I. INTRODUCTION

The calculation of the properties of unsteady boundary layers is important to a wide range of applications including the blades of compressors, turbines and helicopters. In each of these cases, the wakes of previous blades generate a freestream velocity for a subsequent blade with regular, time-dependent fluctuations in amplitude and perhaps frequency. In the helicopter arrangement, variable pitch also leads to a phase relationship between the blade surface and the freestream flow. Discrete-frequency fluctuations of this type influence the characteristics of boundary-layer flows at all Reynolds numbers. The present report is concerned with a method, previously used by Cebeci [2], to allow the efficient and accurate calculation of two-dimensional, boundary-layer flows in the presence of a freestream which varies in amplitude at uniform frequencies. The calculation method is, however, general and is readily capable of extension to include freestream velocity distributions which vary in amplitude and frequency, albeit over a limited range of frequencies; it can also be extended to represent the time-dependent flow over arbitrary three-dimensional bodies.

The solution of the two-dimensional, boundary-layer equations with constant viscosity, an unsteady convective term and a prescribed and unsteady longitudinal pressure gradient is not novel and corresponding calculations are included here to allow the capabilities of the present numerical procedure to be compared with alternatives. McCroskey [3], in a recent and comprehensive review, has suggested that the earlier reviews of Riley [4] and Patel [5] provide an essentially complete theoretical understanding of unsteady laminar flow in the absence of strong pressure gradients. The present method can be used to generate solutions for strong pressure-gradient situations, if required and, with the aid of the Mechul-function approach of reference 6, to calculate through small regions of separated flow. In flows where the direction of velocity reverses and solutions can be obtained without inverse methods, the zig-zag modification to the Box scheme used by Cebeci [7] to calculate the properties of the laminar flow over a cylinder started impulsively from rest, is likely to be necessary.

An important emphasis of the present work is on the application of an efficient, accurate and general numerical procedure to the calculation of unsteady boundary-layer flows. Accuracy, with efficiency, is necessary if practical

calculations are to be performed through regions of flow separation or over three-dimensional geometries and the present two-dimensional calculations provide a basis for the more general purposes.

In formulating equations for turbulent flow which are similar in form to those for laminar flow, it is presumed that the eddy viscosity represents all turbulent viscous effects. The unsteady effects of the freestream are superimposed on the turbulence spectrum through the time-dependent convective term. Alternative, but invariably more complicated, approaches are possible. For example, turbulence energy equations can be solved with frequency or wave number as dependent variables or the subgrid-scale modelling approach can be used with solutions of time-dependent equations only at the lower frequencies. The present eddy-viscosity formulation has the advantage of simplicity and has a wide range of applicability. It is likely, however, that higher-order turbulence models will be required to represent flows with strong favorable and adverse pressure gradients even though the present method can be adequate for some separating boundary layers as shown here.

The report has been prepared with three main sections describing, respectively, the calculation method, the computer program and sample calculations. The description of the calculation method is brief since the equations, the turbulence model, coordinate transformations and solution procedure have been discussed in detail, for example in reference 2. The computer program is described in sufficient detail to allow it to be used and the sample calculations are also described in a manner which is intended to assist a user. The results presented, as a consequence of the sample calculations, demonstrate that the program can conveniently lead to precise results for steady and unsteady laminar and turbulent flows and suggest that future extensions to include free-stream velocities which vary in amplitude and frequency, to separated flow and to three-dimensional flow can readily be achieved. There is, however, an inadequate range of experimental data against which to evaluate the calculations. The report ends with a brief discussion and summary conclusions.

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II. DESCRIPTION OF THE METHOD

2.1 Boundary-Layer Equations in Physical Variables

The governing boundary-layer equations for incompressible unsteady laminar and turbulent boundary layers are:

Continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Momentum

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x} + \frac{1}{\rho} \frac{\partial}{\partial y} \left[\mu \frac{\partial u}{\partial y} - \rho \overline{u'v'} \right] \quad (2)$$

and are considered here with the following boundary conditions for the wall and for the outer edge of the boundary layer:

$$y = 0 \quad u, v = 0 \quad (3a)$$

$$y \rightarrow \infty \quad u \rightarrow u_e(x, t) \quad (3b)$$

To complete the formulation of the problem, initial conditions must be specified in the (t, y) plane for $x = x_0$ and in the (x, y) plane for $t = t_0$. Here we consider a flow in which at time $t = 0$, the flow field is given by steady-state conditions; and, for $t > 0$, the external velocity $u_e(x, t)$ fluctuates from the steady-state velocity $u_0(x)$ according to the expression

$$u_e(x, t) = u_0(x)(1 + B \cos \omega t) \quad (4)$$

At time $t = 0$, the flow can be either laminar or turbulent. In the latter case, the surface distance x must be greater than zero. The initial conditions in the (t, y) plane at $x = 0$ are obtained from the similarity solutions of (1) to (3) for laminar flows as described in detail in reference 2 and later in this report.

2.2 Turbulence Model

The present calculations use the eddy-viscosity concept to model the Reynolds shear stress term of equation (2). According to the formulation described in

reference [2], the eddy viscosity is defined by

$$-\rho \overline{u'v'} = \rho \epsilon_m \frac{\partial u}{\partial y} \quad (5)$$

with two separate formulas used to represent the inner and outer regions of the boundary layer. In the so-called inner region of the boundary layer, ϵ_m is defined by the formula

$$(\epsilon_m)_i = \{0.4y [1 - \exp(-y/A)]\}^2 \left| \frac{\partial u}{\partial y} \right| \quad (6)$$

where

$$A = 26\nu u_\tau^{-1} [1 - 11.8(p_t^+ + p_x^+)]^{-1/2}, \quad u_\tau = \left(\frac{\tau_w}{\rho} \right)^{1/2}, \quad (7)$$

$$p_t^+ = \frac{\nu}{u_e^3} \frac{\partial u_e}{\partial t} \quad \text{and} \quad p_x^+ = \frac{\nu u_e}{u_\tau^3} \frac{du_e}{dx}$$

In the outer region, ϵ_m is defined by

$$(\epsilon_m)_o = \alpha \int_0^\infty (u_e - u) dy \quad (8)$$

where

$$\alpha = 0.0168 \frac{1.55}{1 + \pi}, \quad \pi = 0.55 [1 - \exp(-0.243z_1^{1/2} - 0.298z_1)] , \quad (9)$$

and

$$z_1 = R_\theta / 425 - 1$$

The inner and outer regions are established by the continuity of the eddy-viscosity formulas.

2.3 Governing Equations in Transformed Variables

Before the governing system of equations described above is solved by the numerical method to be described in the following section, they are first expressed in transformed variables which are defined by

$$x = x, \quad t = t, \quad \eta = \left[\frac{u_o(x)}{\nu x} \right]^{1/2} y \quad (10a)$$

In addition, a dimensionless stream function $f(x,t,\eta)$ defined by

$$\psi = (\nu x u_0)^{1/2} f(x,t,\eta) \quad (10b)$$

is used to satisfy the continuity equation and to express the momentum equation (2) as a third-order equation. Here u_0 is some reference velocity and ψ is the stream function defined by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (11)$$

With equations (10) and (11) and with the definition of eddy viscosity, the continuity and momentum equations, (1) to (2), and their boundary conditions (3) may be written as

$$(bf''')' + \frac{P+1}{2} ff'' - P(f')^2 + P_3 = x \left(f' \frac{\partial f'}{\partial x} - f'' \frac{\partial f}{\partial x} + \frac{1}{u_0} \frac{\partial f'}{\partial t} \right) \quad (12)$$

$$\eta = 0; \quad f = f' = 0, \quad \eta \rightarrow \eta_\infty, \quad f' = u_e/u_0 \quad (13)$$

Here primes denote differentiation with respect to η and

$$f' = \frac{u}{u_0}, \quad P = \frac{x}{u_0} \frac{du_e}{dx}, \quad P_3 = \frac{x}{u_0^2} \left(u_e \frac{\partial u_e}{\partial x} + \frac{\partial u_e}{\partial t} \right), \quad b = 1 + \epsilon_m^+, \quad \epsilon_m^+ = \frac{\epsilon_m}{\nu} \quad (14)$$

The initial conditions at $x = 0$ in the (η, t) plane for a laminar flow are obtained by writing (12) as

$$f''' + \frac{P+1}{2} ff'' - P(f')^2 + P_3 = \frac{x}{u_0} \frac{\partial f'}{\partial t} \quad (15)$$

Note that, although for a flat-plate flow the right-hand side of (15) is equal to zero, for stagnation-point flow ($u_0 = Ax$) it is not. For this reason, the slope $A(\equiv du_e/dx)$ must be specified for flows which start with a stagnation point.

The initial conditions at $t = 0$ in the (η, x) -plane for a laminar or turbulent flow are obtained by writing equation (12) as

$$(bf''')' + \frac{P+1}{2} ff'' - P(f')^2 + P_3 = x \left(f' \frac{\partial f'}{\partial x} - f'' \frac{\partial f}{\partial x} \right) \quad (16)$$

Equation (16) expresses conservation of momentum in transformed variables for steady-state conditions. Here P_3 is given by

$$P_3 = \frac{x}{u_0^2} u_e \frac{du_e}{dx} \quad (17)$$

In terms of transformed variables, the eddy viscosity formulas become

$$(\epsilon_m^+)_{\eta} = 0.16 R_x^{1/2} \eta^2 |f''| [1 - \exp(-\eta/A)]^2 \quad (18a)$$

$$(\epsilon_m^+)_{\infty} = \alpha R_x^{1/2} [f'_{\infty} \eta_{\infty} - f'_{\infty}] \quad (18b)$$

where the subscript ∞ refers to the boundary-layer edge and the parameters R_x and η/A are given by

$$R_x = \frac{u_0 x}{\nu}, \quad \frac{\eta}{A} = \frac{R_x^{1/4} (f''_w)^{1/2}}{26} \eta [1 - 11.8(p_t^+ + p_x^+)]^{1/2} \quad (19)$$

2.4 Solution Procedure

We use the numerical method of reference 1 to solve the system of equations described in the previous sections. This is an efficient two-point finite-difference method developed by Keller and Cebeci [1] and extensively used by Cebeci for two-dimensional and three-dimensional flows (see for example, references 8 and 9). A detailed description is presented in references 8 and 10 and is not repeated here.

One of the advantages of the numerical method is that nonuniform net spacings can be used in the t, x directions as well as across the boundary layer. In the latter case, the nonuniform grid is a geometric progression with the property that the ratio of lengths of any two adjacent intervals is a constant; that is, $\Delta \eta_j = K \Delta \eta_{j-1}$. The distance to the j -th line is given by the following formula:

$$\Delta \eta_j = \Delta \eta_1 (K^j - 1) / (K - 1) \quad K > 1 \quad (20)$$

There are two parameters in equation (20): $\Delta \eta_1$, the length of the first step, and K , the ratio of two successive steps. The total number of points J can be calculated from the following formula:

$$J = \frac{\ln[1 + (K - 1)(\eta_\infty/\Delta\eta_1)]}{\ln K} \quad (21)$$

In the computer program which embodies the present solution method, $\Delta\eta_1$ and K are chosen with typical values, for moderate Reynolds numbers, of 0.01 and 1.3, respectively. In general, approximately 50 grid nodes across the boundary layer are sufficient to represent laminar and turbulent boundary-layer flows and, for this reason, the present computer program restricts the number of nodes across the boundary layer to 61. Consequently, the chosen values of $\Delta\eta_1$ and K must be such that the formula which generates the number of grid nodes according to a given or estimated η_∞ , i.e. equation (21), does not allow J to exceed 61. Figure 1 is provided, therefore, to provide guidance in the selection of J .

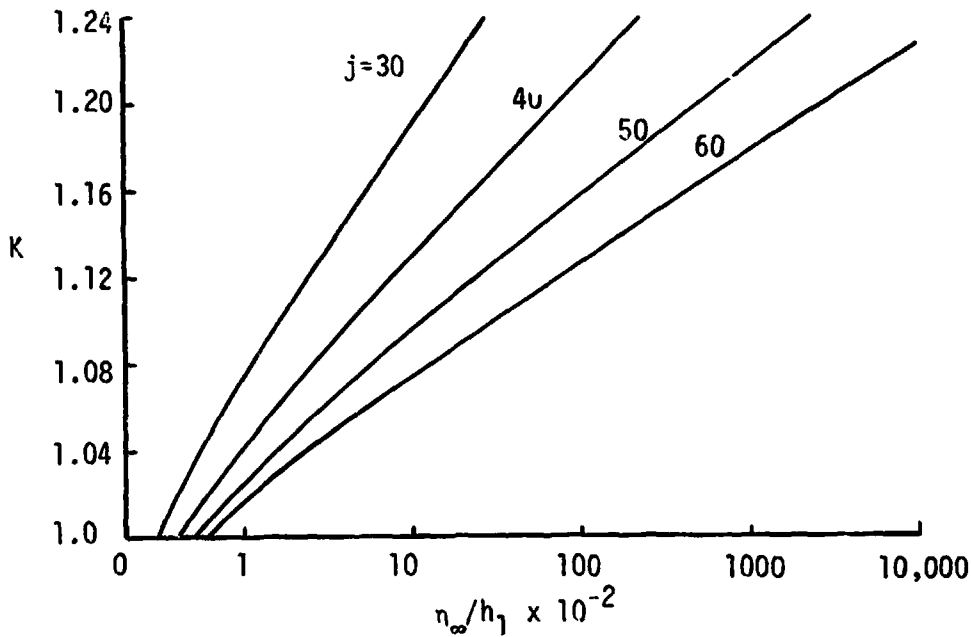


Figure 1. Variation of K with $\Delta\eta_1$ for different η_∞ -values.

In the present program we march along the t-direction. For a specified x-location, the governing equations are solved for each specified t-station. Since a linearized form of the equations is being solved, we iterate at each t-station until some prescribed convergence criterion is satisfied. For both laminar and turbulent flows we use the wall-shear parameter f''_w as the basis for the convergence criterion. For laminar flows the calculation is terminated when

$$|\delta f''_w| < 10^{-4} \quad (22a)$$

For turbulent flows the error in f''_w is expressed as a percentage and the calculations terminated when

$$\frac{2\delta f''_w}{\delta f''_w + 2f''_w} < 10^{-1} \quad (22b)$$

In the present method, the flow is laminar at the leading edge, starting either as a flat-plate flow or as a stagnation-point flow, and can become turbulent at any specified x-station greater than zero. When the transition location is not known experimentally, Michel's transition correlation formula given in reference 6 is used. According to this formula, transition is predicted by the expression

$$R_{\theta_{tr}} = 1.174[1 + (22,400/R_x)]R_x^{0.46} \quad (23)$$

$$0.1 \times 10^6 \leq R_x \leq 40 \times 10^6$$

Previous comparisons with experimental data have shown that this formula works well for steady incompressible flows.

For a given time-dependent external velocity distribution, according to equation (4), and geometric configuration, the present computer program determines the profiles of f , f' , f'' , b as well as the axial distributions of the boundary-layer parameters δ^* , θ , c_f , R_{δ^*} , R_θ , R_x , H . It also computes the reduced frequency $\tilde{\omega}$, the phase angles between wall shear and external velocity, the phase angle between displacement thickness and external velocity for both laminar and turbulent flows, the in-phase and out-of-phase components

of an oscillating turbulent flow. The basic details of the computational procedure is described in the next two sections.

III. DESCRIPTION OF THE COMPUTER PROGRAM

3.1 Input

Essentially the input to the computer program consists of five types of cards. Card 1, shown below, contains the title of the flow problem under consideration.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
TITLE																																																																															

Load Sheet for Card 1

The input is punched as an 80-column alphanumeric field.

Card 2 requires the following information to be specified. The input is in 10I3 format.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
NXT			NZT			NTR			IBDY			ISD			IP2			INTR			KPH4			IPOP			N		

Load Sheet for Card 2

- NXT Total number of t-stations to be calculated
- NZT Total number of x-stations to be calculated
- NTR x-station where transition begins. For all laminar flows, or flows for which transition is to be calculated input
NTR > NZT.
- IBDY Specifies whether the flow at $x = 0$ starts as a flat-plate flow or as a stagnation-point flow.
=1 flat-plate flow
=2 stagnation-point flow

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- ISD Surface distance flag. If surface distance is calculated the pressure gradient parameter P denoted by P_2 is also calculated from the external velocity distribution specified in card 5.
 =1 surface distance calculated
 =2 surface distance input
- IP2 P_2 flag: This option allows the pressure gradient parameter to be either input or calculated from the given external velocity distribution. If ISD=1, IP2 must also be equal to 1.
 =1 P_2 calculated
 =2 P_2 input
- INTR Transition flag
 =1 transition calculated according to (23)
 =2 transition input
- KPHA Phase angle flag. This flag provides the calculation of phase angle between wall shear and external velocity and phase angle between displacement thickness and external velocity for both laminar and turbulent flows.
 =0 phase angle is not calculated
 =1 phase angle is calculated
- IPOP Flag for in-phase and out-of-phase components of an oscillating turbulent flow on a flat plate.
 =0 not computed
 =1 computed
- N Number of t-stations in one cycle to perform the phase angles or in-phase angles or out-of-phase components.

Card 3 requires the following information to be specified. The input is in 7F10.0 format.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
ETA E										DETA(1)										VGP										UINF									

Load Sheet for Card 3

41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70
CB										OMEGA										CA									

Load Sheet for Card 3 (continued)

- ETAE Transformed boundary-layer thickness, η_∞
- DETA(1) Initial $\Delta\eta$ spacing, $\Delta\eta_1$
- VGP Variable grid parameter, K
- UINF Reference freestream velocity, u_∞ (ft/sec)
- CB Amplitude of the fluctuating external velocity B
- OMEGA Radial frequency ω (rad/sec)
- CA Slope of velocity at the stagnation point of an airfoil, $(du_0/dx)_{x=0}$

Card 4 contains the input t-stations which are read in F10.0 format.

1	2	3	4	5	6	7	8	9	10
X									

Load Sheet for Card 4

Card 5 contains the geometry of the two-dimensional body and the steady-state external velocity distribution $u_0(x)$, or the dimensionless pressure gradient parameter $P(x)$ in 3F10.0 format. The geometry of the body is either read in by specifying the surface distance of the body or is computed from the (x/c) and (y/c) coordinates of the body. Here c is a reference length of the body, which for an airfoil is usually taken as the chord. The computer names for the usual (x/c) , (y/c) coordinates are denoted by ZC and YC, respectively. Note that when surface distance denoted by Z in field 1, is to be calculated, then the third field must contain the external velocity distribution from which the pressure gradient P2 must be calculated. If surface distance Z is input, and P2 is to be calculated, the third field may be blank. There will be NZT cards of this type.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
ZC or Z										YC or U0										U0 or P2									

L ad Sheet for Card 5

3.2 Output

Station header

NX number of t-station
 NZ number of x-station
 X t, time coordinate
 Z x, space coordinate

Iteration print

V(WALL) f''_w
 DELVW $\Delta f''_w$

Transition values — if INTR=1 (Card 2), these values will be printed close to transition

RTHETA R_θ , determined from the boundary-layer calculations.
 RTHT $(R_\theta)_{tr}$, computed according to formula (23) for a given R_v .

If transition occurs and a station is interpolated, we print out new values of P2, U0 and Z which correspond to P, U_0 and x.

The output boundary-layer parameters include profiles f , f' , f'' and b as a function of the similarity variable η and grid parameter j . Here

ETA η
 F f'
 U f'
 V f''
 B $b (=1 + \epsilon_m^+)$ equals 1.0 for laminar flows

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They also include displacement thickness δ^* , momentum thickness θ , local skin-friction coefficient c_f , Reynolds numbers based on δ^* , θ and x , that is, R_{δ^*} , R_θ and R_x shape factor H . The definition of these parameters and their computer notation is

$$\text{DELSTR} \quad \delta^* = \int_0^{\infty} (1 - u/u_e) dy$$

$$\text{THETA} \quad \theta = \int_0^{\infty} u/u_e (1 - u/u_e) dy$$

$$\text{CF} \quad c_f = 2\tau_w/\rho u_0^2$$

$$\text{RDELST} \quad R_\delta^* = \delta^* u_0 / \nu$$

$$\text{RTHETA} \quad R_\theta = \theta u_0 / \nu$$

$$\text{RZ} \quad R_z = -u_0 / \nu$$

$$\text{H} \quad \delta^* / \theta$$

In terms of transformed variables, δ^* , θ and c_f can be written as

$$\delta^* = \frac{z}{\sqrt{R_z}} [\eta_\infty - f_\infty / f'_\infty]$$

$$\theta = \frac{z}{\sqrt{R_x}} \int_0^{\eta_\infty} \frac{f'_\infty}{f'_\tau} (1 - \frac{f'_\tau}{f'_\infty}) d\eta$$

$$c_f = 2 \frac{f''_w}{\sqrt{R_z}}$$

In addition to the above data, the computer program also prints out the reduced frequency $\tilde{\omega}$ ($\equiv \omega x / u_0$), the phase angle between wall shear and external velocity, phase angle between displacement thickness and external velocity for both laminar and turbulent flows. For turbulent flows it also computes the in-phase and out-of-phase components of an oscillating turbulent flow on a flat plate c , described in section 4.4.

IV. SAMPLE CALCULATIONS

The present computer program can be used to determine the properties of two-dimensional laminar and turbulent boundary layers for steady and unsteady flows. In order to demonstrate its capability and its input requirements, a number of sample calculations are presented in the following subsections.

4.1 Steady Laminar Flows

The nonsimilar laminar flow for which the inviscid velocity distribution is given by

$$u_0 = u_\infty \left(1 - \frac{1}{8} x\right) \quad (24)$$

is commonly referred to as Howarth's flow and has been computed extensively and accurately. Since this is a steady flow, we choose $NXT=1$. According to previous calculations, flow separation takes place at $x = 0.96$ and thus, by choosing Δx to be uniform and equal to 0.05, there are 21 x-stations to $x = 1.0$. Consequently, we set $NZT=21$. By setting NTR greater than NZT , say 50, we avoid the turbulent flow calculations. Since this flow starts as a flat-plate flow ($P2 \equiv 0$), we let $IBDY=1$. We also set $ISD=2$, $INTR=2$ and by choosing $IRZ=1$, we ask the program to compute $P2$. We also set $KPHA$, $IPOP$, N equal to zero.

The transformed boundary-layer thickness is constant for most steady laminar flows and a value of 8 is sufficient. For this reason a value of 8 is read in for $ETA E$ at the first x-station. If needed, for other x-stations, the boundary-layer thickness would grow internally.

In general 40 to 50 node points across the boundary layer are sufficient and, for laminar flows, VGP should be set equal to 1.0. Therefore, by choosing $VGP=1.0$ and $DETA(1)=0.20$, we take 41 points uniformly spaced across the boundary layer. We let $UINF=1.0$. Because this is a steady laminar flow, we set $CB=0$, $OMEGA=0$. Also we set $CA=0$ since the flow starts as a flat-plate flow.

In card 4, we set $t(=x)=0$, and in card 5, since the surface distance is read in, we fill in the 21 x -values as 0, 0.05, 0.10, etc. under z , and the values of U_0 as 0, 0.9937, 0.98750, etc. under U_0 .

Figure 2 shows the computed wall shear parameter $f''(0)$ as a function of x . The results agree well with those reported in the literature, see for example [8]. The wall shear parameter $f''(0)$, which is positive at $x = 0.95$, becomes negative at the next input station, $x = 1.0$, indicating that flow separation has taken place between $0.95 < x < 1.0$. According to the extrapolation shown in figure 1, the flow separation takes place at $x = 0.96$, and this agrees well with other calculations. When $f''(0)$ becomes negative, the calculations are terminated internally.

4.2 Steady Turbulent Flows

To test the computer program for steady turbulent flows, we have chosen three experimental incompressible flows from the data reported at the Stanford Conference on Computation of Turbulent Boundary Layers [11], i.e. the flows identified as 1400, 2100 and 3300.

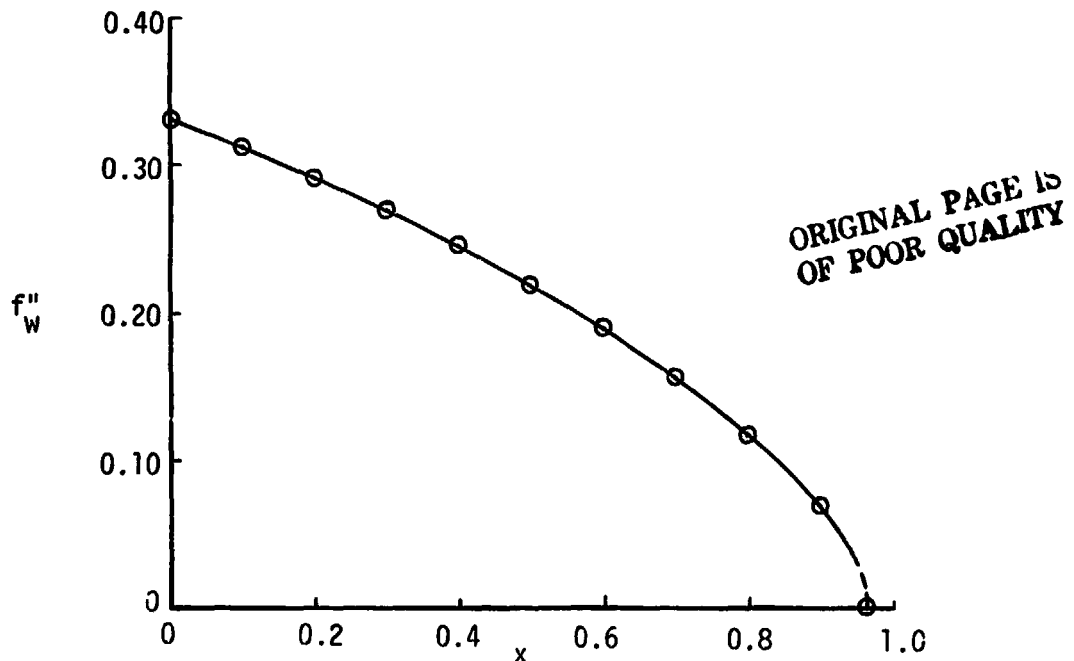


Figure 2. Computed wall shear parameter variation with x for Howarth's flow. The circles denote those computed by Cebeci and Smith [8].

Flow 1400 corresponds to a boundary layer with zero pressure gradient and the experimental data is due to Wieghart. Flow 2100 corresponds to a boundary layer with an initially favorable pressure gradient followed by a zero pressure gradient and an adverse pressure gradient that causes the flow to separate; the experimental data is due to Schubauer and Klebanoff. Flow 3300 is an equilibrium boundary layer with an adverse pressure gradient, and is due to Bradshaw.

In making these calculations we have first computed a zero pressure gradient flow to match the initial conditions provided by the experiment. Then we specified the measured external velocity distribution as a function of x and performed the boundary-layer calculations. In all cases, we have taken $\Delta\eta_1 = 0.01$ and $K = 1.3$.

For flow 1400, an arbitrary x -distribution was selected with $x = 0, 0.01, 0.05, 0.10, 0.25, 0.50, 0.75, 1, 1.6, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 15, 18, 21$ and $u_e = 108$ ft/sec. The transition location was specified at $x = 0.01$ and the calculations performed for $NXT=1$ and $NZT=22$.

Figure 3 allows a comparison of calculated and experimental values of the local skin-friction coefficient, c_f , as a function of R_θ . The agreement between calculated results and experimental results is similar to those shown by Cebeci and Smith calculations for steady flows [8].

For flow 2100, we have chosen a flat-plate of length approximately 2 ft with $u_e = 117$ ft/sec to match the experimental velocity profiles at $x = 2$ ft. Further downstream, the experimental velocity distribution, u_e , was specified as a function of x and the calculations performed for a total of 30 x -stations ($NZT=30$) and for one-time station $t = 0$ ($NXT=1$). Figure 4 allows a comparison of the calculated and experimental shape factor H , Reynolds number based on momentum thickness, R_θ , and local skin-friction coefficient, c_f . As before, the agreement between calculated and experimental results is similar to that demonstrated previously by Cebeci and Smith in steady flow calculations [8].

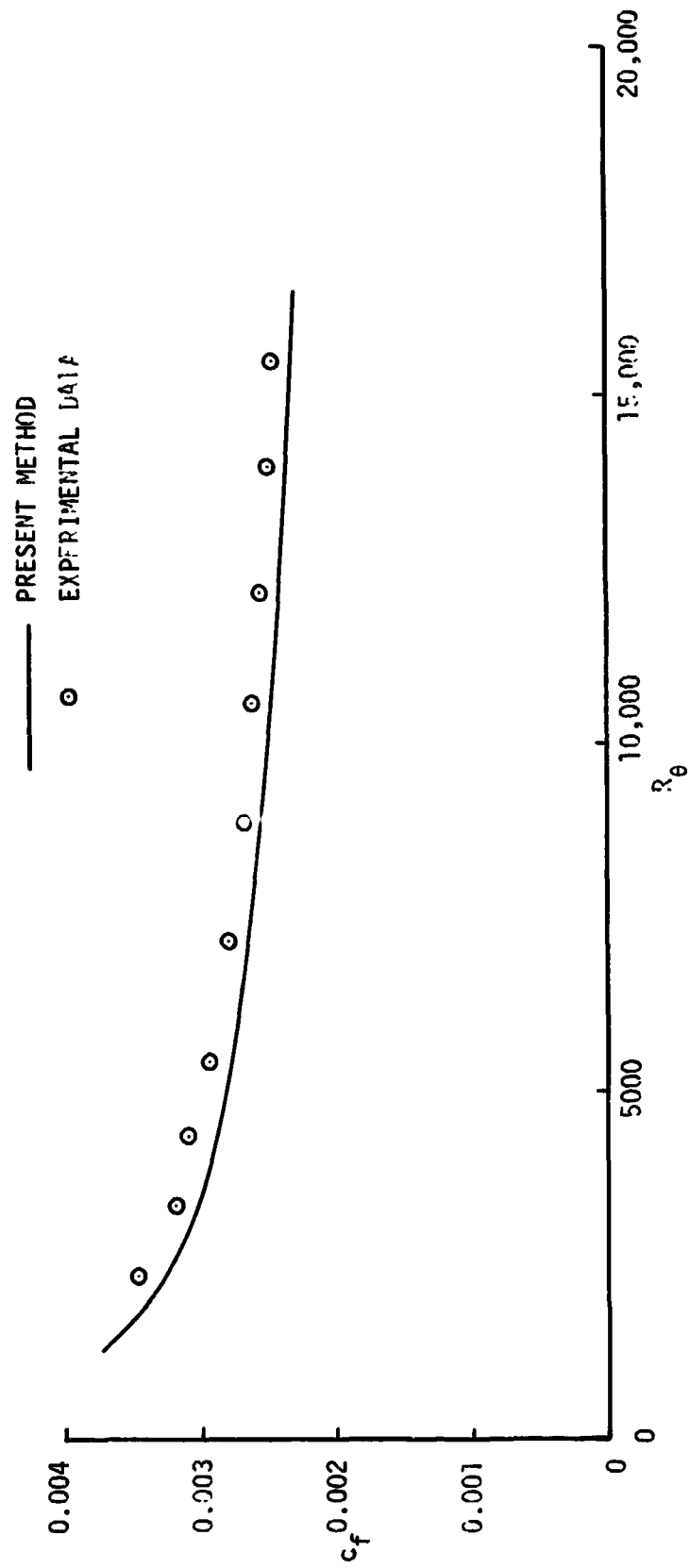


Figure 3. Comparison of calculated and experimental results for flow 1400.

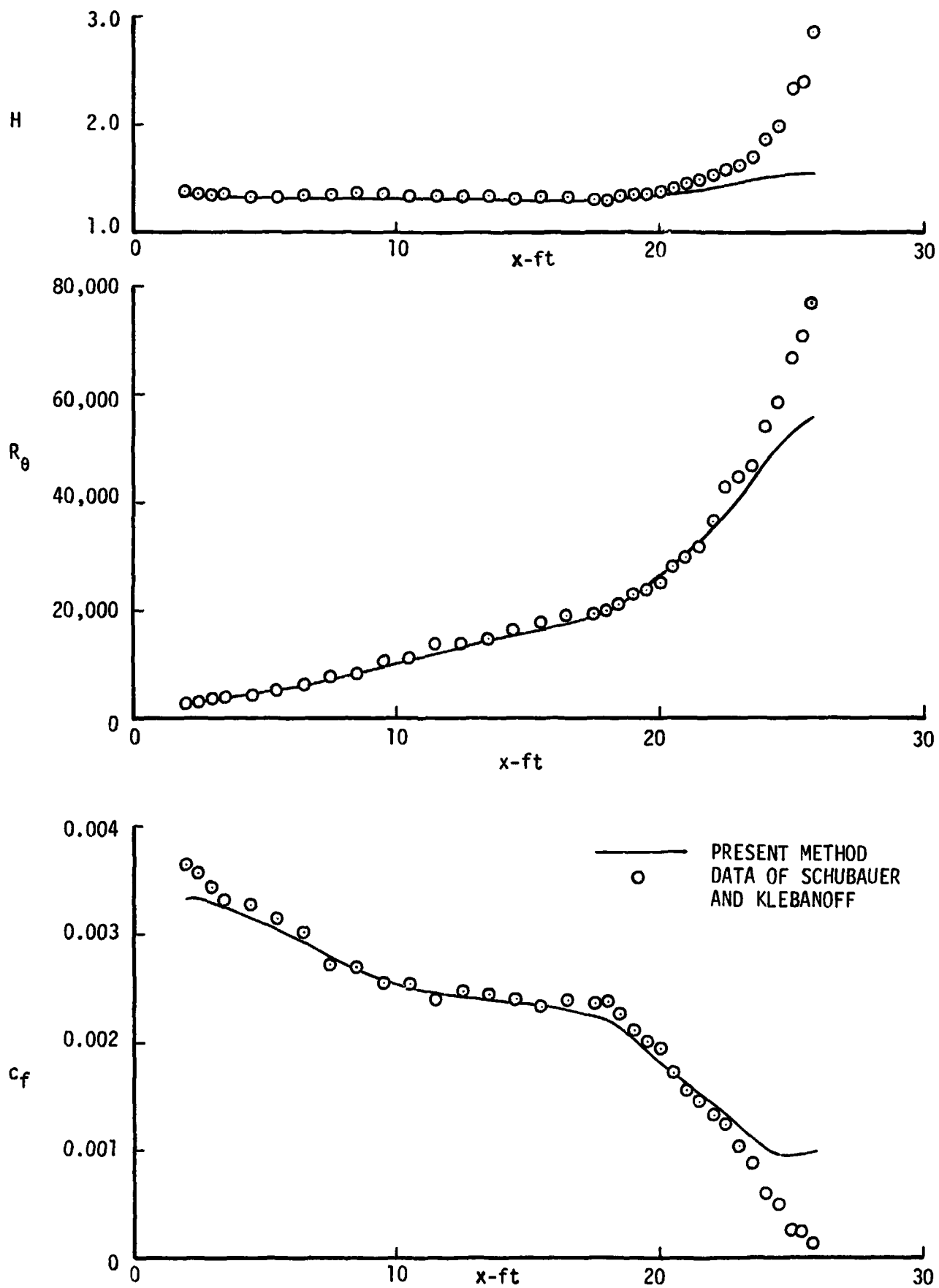


Figure 4. Comparison of calculated and experimental results for flow 2100.

For flow 3300 we have again chosen a flat-plate of length 6 ft and $u_e = 125.63$ ft/sec to match the first experimental velocity profile given by Bradshaw at $x = 2$ ft. After that, the experimental velocity distribution was specified and the calculations performed for a total of 22 x -stations ($NZT=22$), 14 of them corresponding to the flat-plate flow. Figure 5 shows the calculated and experimental shape factor H , Reynolds number based on momentum thickness, R_θ , and local skin-friction coefficient, c_f .

4.3 Unsteady Laminar Flow on a Flat Plate

According to Lighthill's analysis [12], the phase angle (ϕ) between the wall shear and the external velocity distribution for a laminar flow on a flat plate for which the external flow is oscillating according to equation (4) is given by separate formulas depending on whether the reduced frequency $\omega(\equiv \omega x/u_0)$ is much smaller or much greater than unity. In order to test the present method for this flow and with a range of reduced frequencies, the following procedure was adopted to determine the phase angle between $u_e(x_0, t)$ and $f_w''(x_0, t)$. The values of $u_0(x_0)$ and $\overline{f_w''(x_0)}$ are first computed from the expressions

$$u_0(x_0) = \frac{1}{p} \int_{t_0}^{t_0+p} u_e(x_0, t) dt \quad (31a)$$

$$\overline{f_w''(x_0)} = \frac{1}{p} \int_{t_0}^{t_0+p} f_w''(x_0, t) dt \quad (31b)$$

where $p = 2\pi/\omega$ and represents the period of the oscillation of the mainstream.

From equation (4),

$$u_e(x_0, t) - u_0(x_0) = A \cos \omega t \quad (32)$$

where $A \equiv u_0(x_0)B$. Similarly, we can write

$$f_w''(x_0, t) - \overline{f_w''(x_0)} = C \cos[\omega t + \phi(x_0)] = C[\cos \omega t \cos \phi(x_0) - \sin \omega t \sin \phi(x_0)] \quad (33)$$

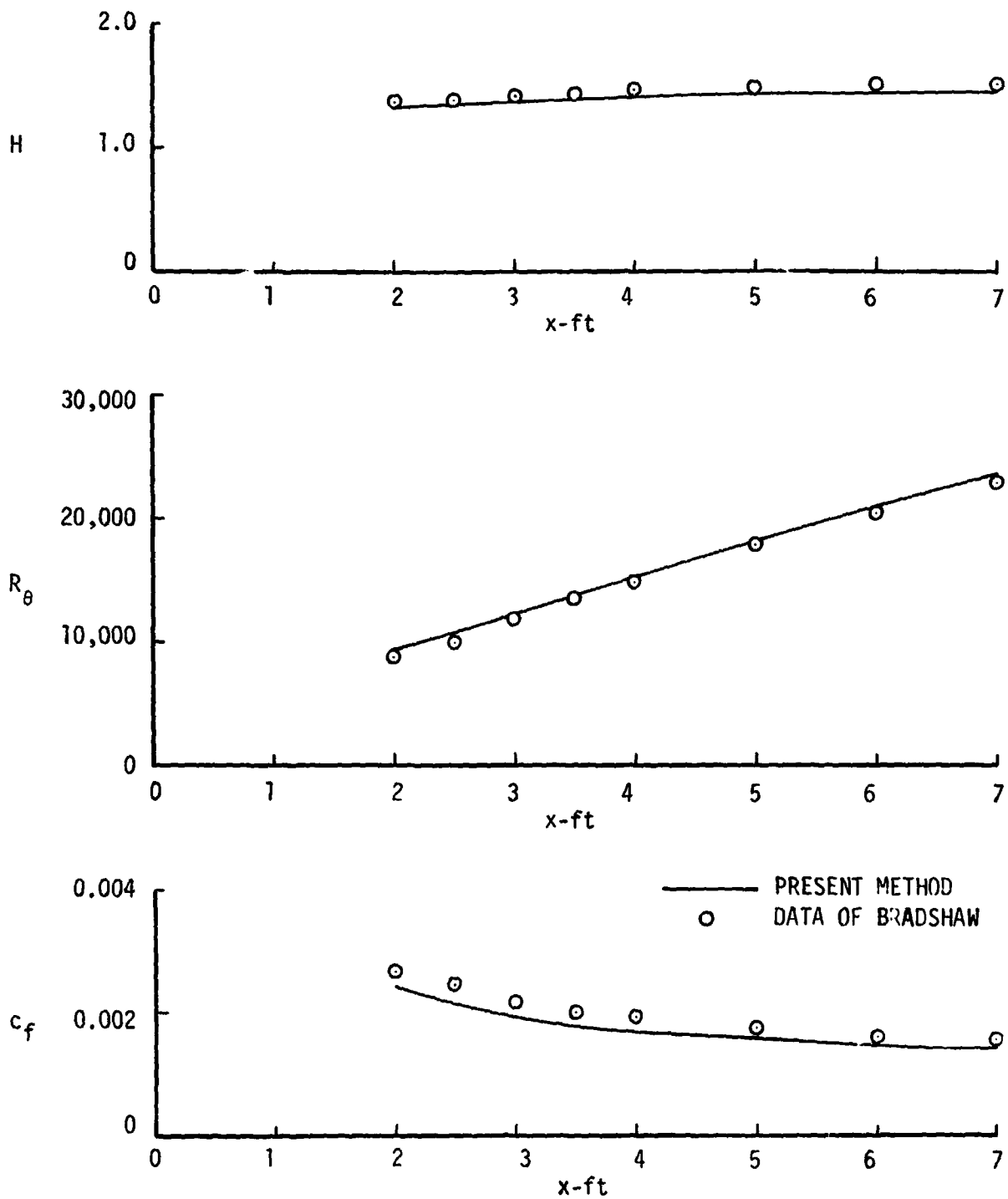


Figure 5. Comparison of calculated and experimental results for flow 3300.

with $\phi(x_0)$ denoting the phase angle between u_e and f_w'' at $x = x_0$.
Integration of the product of equations (32) and (33) leads to

$$\cos\phi(x_0) = \frac{\int_{t_0}^{t_0+p} [u_e(x_0, t) - u_0(x_0)] \cdot [f_w''(x_0, t) - \overline{f_w''(x_0)}] dt}{AC\pi/\omega} \quad (34)$$

with

$$A^2 = \frac{\omega}{\pi} \int_{t_0}^{t_0+p} [u_e(x_0, t) - u_0(x_0)]^2 dt \quad (35a)$$

$$C^2 = \frac{\omega}{\pi} \int_{t_0}^{t_0+p} [f_w''(x_0, t) - \overline{f_w''(x_0)}]^2 dt \quad (35b)$$

In order to perform the calculations for one cycle, we must choose Δt according to the formula

$$\Delta t = \frac{P}{(N-1)} \quad (36a)$$

where N corresponds to the total number of t -stations, that is, NXT . Introducing the definition of p into equation (36a), we get

$$\Delta t = \frac{2\pi}{\omega(N-1)} \quad (36b)$$

and compute

$$t_n = n\Delta t_n \quad n = 1, 2, \dots, N \quad (36c)$$

In our calculations for this case, we have chosen $B = 0.125$, $\omega = 1.57$ rad/sec, $u_0 = 160$ ft/sec, $\Delta\eta_1 = 0.25$, $K = 1.0$ and computed Δt from (36b). For $N = 20$, corresponding to one cycle, equation (36b) gives

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$$\Delta t = \frac{2\pi}{1.57(20)} = 0.20$$

Then $t_0 = 0$, $t_1 = 0.20$, $t_2 = 0.40$, ..., $t_N = (21 - 1)\Delta t = 4.0$. If we want to perform the calculations for two cycles, then $N \times T = 41$ with $\Delta t = 0.20$ so that $t_N = 8.0$. For three cycles, $N \times T = 61$ with $\Delta t = 0.2$ and $t_N = 12.0$.

Figure 6 presents results obtained from the present procedure and those predicted by Lighthill's analysis: as can be seen, the results are in close agreement at the two asymptotes. The results are also in good agreement with the numerical computations of Ackerberg and Phillips [13]. They show that if $B \leq 0.125$, Lighthill's low-frequency approximation to the phase angle is satisfactory for $\tilde{\omega} < 0.2$ and his high-frequency approximation is satisfactory for $\tilde{\omega} \geq 2.6$.

The calculations shown in Table 1 also indicate that it is desirable to compute for two periods in order to determine the phase angle accurately. The results of figure 6 were obtained by computing two periods. Additional calculations indicated that the phase angles computed for three periods did not differ from those computed for two periods.

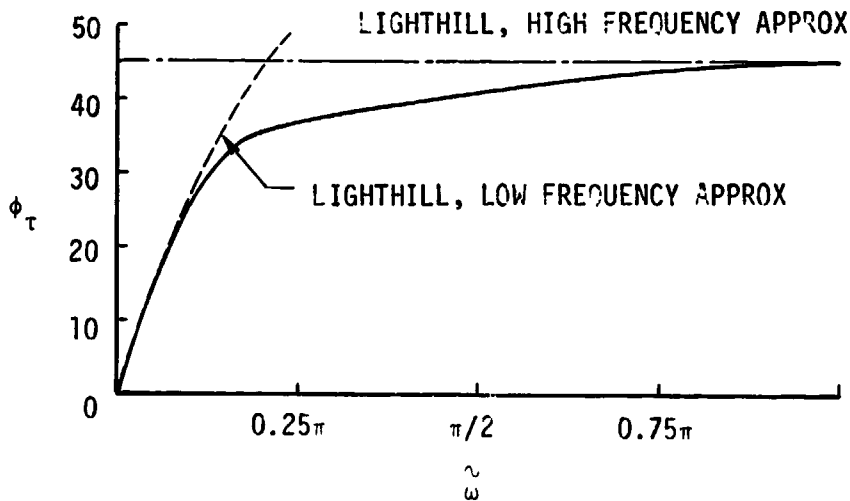


Figure 6. Computed phase angle between wall shear and external velocity for a laminar flat-plate flow.

Table 1. Computed Phase Angles Between Wall Shear and External Velocity According to (4)

$\frac{\omega}{u_e}$	ϕ_τ One Cycle	Two Cycles
0.157	14.64	14.64
0.314	25.40	25.30
0.477	32.25	31.79
0.628	36.28	35.20
0.785	38.86	36.99
0.942	40.60	37.91
1.099	42.00	38.58
1.256	43.13	39.20
1.413	44.13	39.95
1.57	44.96	40.77
1.727	45.70	41.64
1.884	46.31	42.45
2.041	46.85	43.18
2.198	47.29	43.76
2.355	47.67	44.23
2.512	47.98	44.57
2.669	48.26	44.82
2.826	48.49	44.98
2.983	48.70	45.09
3.140	48.87	45.13

The procedure used for calculating the phase angle between the wall shear and the external velocity distribution can also be used to compute the phase angle between the displacement thickness, δ^* ,

$$\delta^* \equiv \int_0^\infty (1 - \frac{u}{u_e}) dy$$

and the external velocity. This is achieved by replacing equation (31b) by

$$\overline{\delta^*(x_0)} = \frac{1}{p} \int_{t_0}^{t_0+p} \delta^*(x_0, t) dt$$

and modifying equations (33), (34) and (35b) in a similar way. In general, at low frequencies and at high frequencies, the computed phase angles for one cycle and for two cycles are nearly the same (see Table 2). At moderate frequencies the ones computed by using two cycles differ from those

Table 2. Computed Phase Angles Between Displacement Thickness and External Velocity According to (4)

$\tilde{\omega}$	ϕ_{δ^*}	
	One Cycle	Two Cycles
0.157	169.56	169.57
0.314	163.03	163.16
0.477	160.06	160.71
0.628	159.70	161.20
0.785	160.73	163.42
0.942	162.35	166.50
1.099	164.12	169.85
1.256	165.87	173.06
1.413	167.50	175.80
1.57	168.99	177.58
1.727	170.35	177.68
1.884	171.57	176.88
2.041	172.66	176.18
2.198	173.60	175.76
2.355	174.37	175.60
2.512	174.92	175.56
2.669	175.30	175.53
2.826	175.49	175.42
2.983	175.49	175.05
3.140	175.40	174.47

obtained by using one cycle. The difference is probably due to the truncation error and can be avoided by taking more points across the boundary layer.

Figure 7 presents the computed phase angle results, using one cycle for this case. It is seen that at about the same reduced frequency $\tilde{\omega} = 3.0$ for which ϕ_{τ} reaches its maximum value of 45° , ϕ_{δ^*} reaches its maximum value of 175.5° (one cycle).

4.4 Unsteady Turbulent Flow on a Flat Plate

An unsteady turbulent boundary-layer flow on a flat plate is considered in this subsection which indicates how the computer program can be used to calculate the flow characteristics corresponding to the configuration of Karlsson [14]. The procedure for computing the in-phase and out-of-phase components of an oscillating turbulent flow over a flat plate is also outlined. For simplicity,

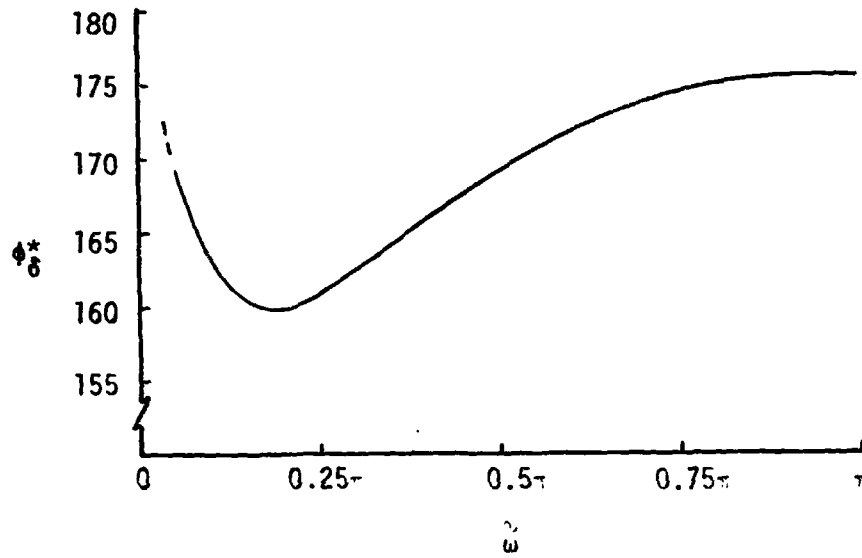


Figure 7. Computed phase angle between displacement thickness and external velocity for a laminar flat-plate flow.

Karlsso's notation is used and denotes the x-component of the velocity within the boundary layer by

$$u(x,y,t) = \bar{u}(x,y) + u^{(1)} \cos\phi \cos\omega t - u^{(1)} \sin\phi \sin\omega t \quad (37)$$

Since $f' = u/u_0$, this expression can be written as

$$u_0 f' = \bar{u}(x,y) + u^{(1)} \cos\phi \cos\omega t - u^{(1)} \sin\phi \sin\omega t \quad (38)$$

Using equation (4) and denoting $u_0 B = u_\infty^{(1)}$, equation (38) can be written as

$$\frac{u}{u_\infty^{(1)}} = \frac{f'}{B} = \frac{\bar{u}}{u_0 B} + \frac{u^{(1)} \cos\phi}{u_\infty^{(1)}} \cos\omega t - \frac{u^{(1)} \sin\phi}{u_\infty^{(1)}} \sin\omega t \quad (39)$$

To compute the in-phase component $u^{(1)} \cos\phi / u_\infty^{(1)}$ from equation (39) we multiply both sides of that expression by $\cos\omega t$ and integrate with respect to t from 0 to $2\omega/\pi$ to obtain

$$\frac{u^{(1)} \cos\phi}{u_\infty^{(1)}} = \frac{\omega}{\pi} \frac{1}{B} \int_0^{2\pi/\omega} f'(\eta, t) \cos\omega t dt \quad (40)$$

Similarly, we multiply both sides by $\sin\omega t$ and integrate to determine the out-of-phase component, i.e.

$$\frac{u^{(1)}}{u_\infty^{(1)}} \sin \phi = -\frac{\omega}{\pi} \frac{1}{B} \int_0^{2\pi/\omega} f'(n,t) \sin \omega t dt \quad (41)$$

As in the phase-angle calculations, the in-phase and out-of-phase component of the oscillating flow must be computed with uniform steps in Δt as discussed in section 4.3. We approximate equations (40) and (41) by [recalling $(f')_j^n = f'(n_j, t_n)$]

$$\frac{u^{(1)}}{u_\infty^{(1)}} \cos \phi = \frac{2}{B(N-1)} \sum_{n=0}^{N-1} (f'_j)^n \cos \omega t_n \quad (42)$$

$$\frac{u^{(1)}}{u_\infty^{(1)}} \sin \phi = -\frac{2}{B(N-1)} \sum_{n=0}^{N-1} (f'_j)^n \sin \omega t_n \quad (43)$$

Figure 8 shows the calculated values of the in-phase and out-of-phase components for $u_\infty^{(1)}/u_0 = 14.7\%$ and $\omega = 25.133$ radians/sec or $\omega/2\pi = 4$ cycles/sec. These calculations were started as laminar at $x = 0$ for the external flow given by equation (4) with $u_0 = 17.5$ ft/sec. The Δx -spacing for turbulent flows was one foot. The turbulent flow calculations were started at $x = 0.01$ ft and the experimental velocity profile matched at $x = 12.5$ ft (see fig. 6 of [7]). Altogether a total of 14 x -stations and 25 t -stations corresponding to two cycles were used. Computed results for one cycle and for two cycles show that for all practical purposes the results are the same.

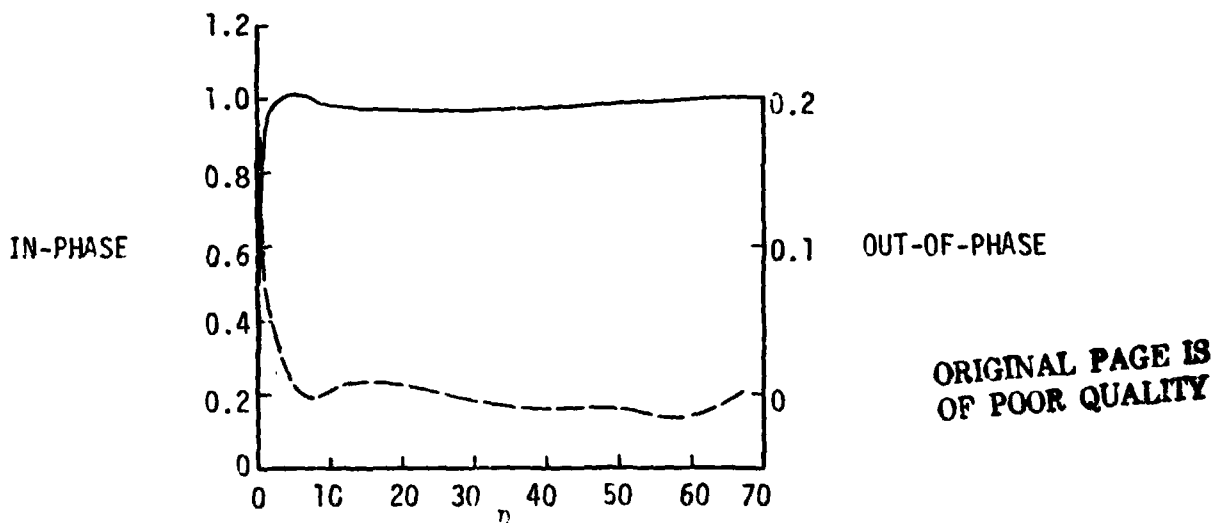


Figure 8. Computed in-phase (—) and out-of-phase (---) components of an oscillating turbulent flow for Karlsson's data.

Table 3. Computed Phase Angles Between Wall Shear and External Velocity According to Equation (4) for Turbulent Flows, $\omega/2\pi = 4$ cycles/sec, $B = 14.7\%$.

$\frac{\omega}{\omega}$	ϕ_{τ}	
	One Cycle	Two Cycles
11.49	21.83	20.17
12.93	22.44	21.44
14.36	22.92	21.89
15.80	23.36	22.34
17.23	23.61	22.61
18.67	23.87	23.06

According to the computed results, we also observe that the phase angle between the displacement thickness and external velocity show some sensitivity depending on whether one uses one cycle or two cycles as shown in Table 4. However, the difference is relatively small and can probably be avoided by taking more x-stations and more grid points across the boundary layer.

Table 4. Computed Phase Angles Between Displacement Thickness and External Velocity According to Eq. (4) for Turbulent Flows, $\omega/2\pi = 4$ cycles/sec, $B = 14.7\%$.

$\frac{\omega}{\omega}$	ϕ_{δ^*}	
	One Cycle	Two Cycles
11.49	164.92	161.89
12.93	169.57	155.11
14.36	170.13	163.51
15.80	169.93	170.04
17.23	174.08	173.96
18.67	172.23	171.25

4.5 Unsteady Laminar and Turbulent Flow on an Airfoil

To compute a laminar and turbulent flow on an airfoil in which the external velocity distribution varies according to eq. (4), we considered NACA 0012 at zero angle of attack. The pressure distribution for steady flow is given in

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ref. [15]. A total of 25 x-stations and 15 t-stations were used. In the latter case a constant Δt spacing of 0.2 was used with $\omega = 1.57$ rad/sec, $\Delta \eta_1 = 0.05$, $K = 1.2$, $u_\infty = 160$ ft/sec, and $B = 0.125$. The slope of u_0 (du_0/dx) at the stagnation point was determined to be 200/sec. The airfoil geometry was read in as (x/c) , (y/c) and the surface distance calculations for the given $u_0(x)$ distribution were performed internally by setting $ISD = 1$. The transition point was not specified in the input and the computer program was asked to compute it by setting $INTR=1$.

Figure 9 shows the computed local skin-friction coefficient and figure 10 the computed displacement thickness values as a function of x for various values of t . We note that, according to equation (23), transition occurs between the two input x-stations, $x = 0.50027$ and $x = 0.55027$; the interpolated x-station that corresponds to the transition station is $x = 0.5361$. The results in these figures show slight wiggles for turbulent flows. This is due to the neglect of the finite length of the transition region that exists between a laminar and turbulent flow. Although this region is small at low Reynolds number, it is not at high Reynolds number and the step change to turbulent flow represented by the calculations, lead to oscillations. This can be avoided by using an intermittency expression such as that described in [8].

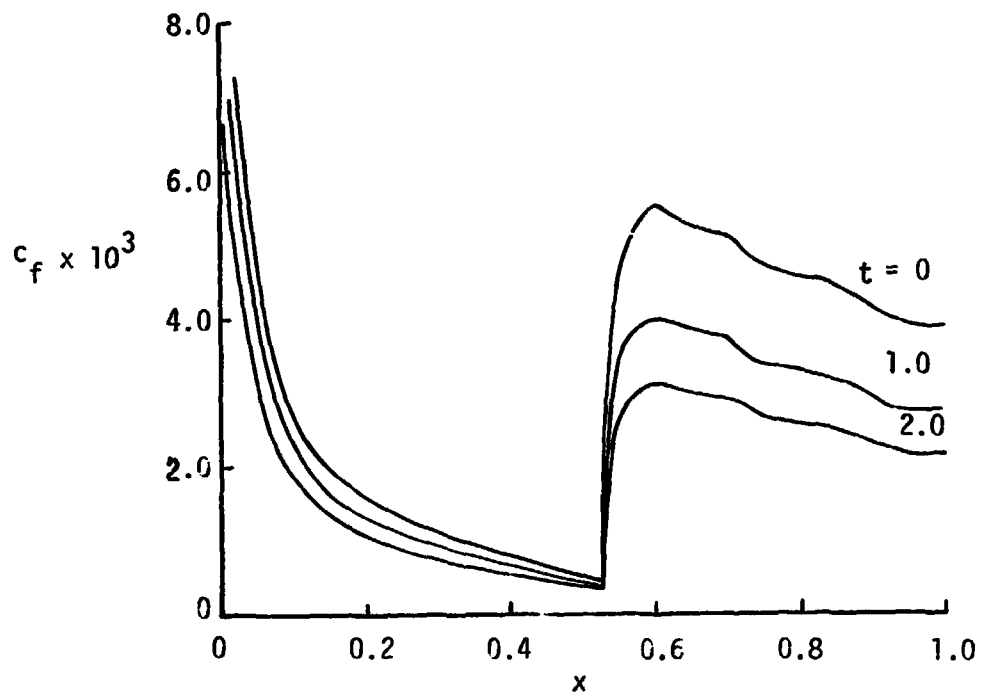


Figure 9. Computed local skin-friction distribution for the NACA 0012 airfoil.

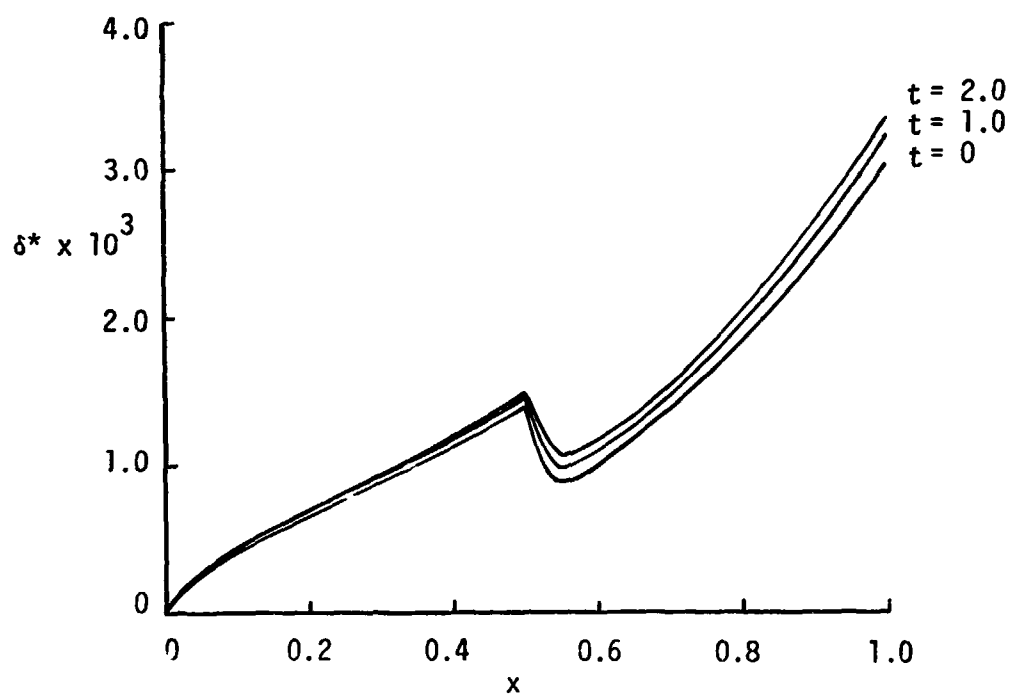


Figure 10. Computed displacement thickness distribution for the NACA 0012 airfoil.

V. DISCUSSION AND SUMMARY CONCLUSIONS

In addition to providing a potential user with a guide to the computer program embodying the present calculation method, the results provide information of current and future value. It is clear from the steady-state solutions, that accurate results can be obtained with an algebraic eddy-viscosity hypothesis for a wide range of pressure gradients including severe adverse gradients which lead to separation. The execution time, on a CDC 6600, for a typical calculation (e.g. that of figure 2) was of the order of 0.09 sec.

The unsteady laminar results indicate that calculations have to be completed through two periods to provide converged and precise results for the phase angles between the wall-shear stress and the freestream velocity and between the displacement thickness and the freestream velocity. As expected, the displacement-thickness phase angle is less amplitude dependent than is the shear-stress phase angle and has considerably larger value. Two cycles are also necessary to calculate phase angles for turbulent flow which are significantly lower than those for laminar flow; the calculated values for shear-stress phase angle agree with available measurements within experimental accuracy. The execution time to obtain the unsteady turbulent results of figures 6 and 7, was approximately 16 sec and suggests that the numerical solution of unsteady, three-dimensional equations is a practical and not excessively expensive proposition.

The airfoil results show that the present procedure may readily be applied to practical configurations with calculations from a leading to trailing edge. Extensions to include the wake and airfoil separation require the incorporation of known techniques and, like extensions to three-dimensional flow, offer no obstacles of principle. The precision of the calculation of transition from laminar to turbulent flow is, however, a major unknown. The review of Loehrke, Morkovin and Fejer [16] suggests, as might be expected, that the transition Reynolds number decreases with the amplitude of freestream fluctuations and is also influenced by their frequency. The results upon which conclusions might be based are not in quantitative agreement but it is probable that Michel's steady-state transition [8] overestimates the transition Reynolds number of up to a factor of two.

The following paragraphs provide a summary of the more important conclusions which may be extracted from the preceding text:

1. The calculation method can be used, in the form embodied in the presently described computer program, to calculate the properties of laminar and turbulent boundary layers with pressure gradient and with a freestream velocity which fluctuates in time with arbitrary amplitude.
2. The times, approximately 0.09 sec and 16 sec of CDC 6600 for steady and unsteady calculations are modest and suggest that three-dimensional calculations can also be performed with acceptable cost.
3. The calculated phase angles between wall-shear stress and freestream velocity increase with amplitude for laminar flow and are greater than the almost constant value of approximately 20 degrees obtained for turbulent flow. The corresponding phase angles between displacement thickness and freestream velocity are around 170 degrees for both laminar and turbulent flow. Two cycles are required to obtain converged solutions.
4. The properties of the boundary layer on an airfoil from the stagnation line through transition into the downstream turbulent boundary layer have also been determined with a varying-amplitude freestream velocity. Experimental information is required to confirm the precision of these calculations and particularly to indicate the influence of freestream fluctuations on transition. The transition correlation of Michel has been used for these calculations but alternatives can readily be incorporated.

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VII. COMPUTER LISTING AND A SAMPLE CALCULATION

In this section we present a listing of the computer programs and to illustrate the use of the computer program we present a sample calculation for a two-dimensional laminar flow discussed in Section 4.1. Although the calculations in that example were done for a total of 21 NZ-stations, we only show the first ten stations to illustrate a typical output of our computer program.

```

COMMON/BLC0/ NXT,NZT,NX,NZ,NTR,NP,ITMAX,INTR,IBDY,KPHA,IPOP,N,
1 NPT,CNU,ETA,E,VGP,A(61),ETA(61),DETA(61)
COMMON/BLC1/ CA,CB,OMEGA,OMX,REDFR,X(41),Z(30),UO(30),RZ(30),
1 P1(30),P2(30),P3(41,30),UE(41,30)
COMMON/PROF/ DELV(61),F(61,41,2),U(61,41,2),V(61,41,2),B(61,41,2)
C - - - - -
CALL INTIAL
WRITE(6,9000)
25 WRITE(6,9100) NX,NZ,X(NX),Z(NZ)
   OMX = OMEGA*X(NX)
30 IT = 0
   IGROW = 0
60 IT = IT+1
   IF(IT .LE. ITMAX) GO TO 65
   WRITE(6,2500)
   STOP
65 IF(NZ .GE. NTR) CALL EDDY
   IF(NZ .GT. 1) GO TO 70
   CALL BCONX
   GO TO 95
70 IF(NX .GT. 1) GO TO 90
   CALL BCONZ
   GO TO 95
90 CALL COEFG
95 CALL SOLV3
   IF(V(1,NX,2) .GE. 0.0) GO TO 61
   IF(NZ .GE. NTR) GO TO 61
   WRITE(6,9300)
   STOP
C CHECK FOR CONVERGENCE
61 IF(NZ .GE. NTR) GO TO 62
C - LAMINAR FLOW
   IF(ABS(DELV(1)) .GT. 0.001) GO TO 60
   GO TO 100
C - TURBULENT FLOW
62 IF(ABS(DELV(1)/(V(1,NX,2)+0.5*DELV(1))) .GT. 0.02) GO TO 60
100 IF(NP .EQ. NPT) GO TO 200
   IF(ABS(V(NP,NX,2)) .LE. 0.001) GO TO 200
   IF(IGROW .EQ. 1) GO TO 200
   IGROW = 1
   LL = 2
   CALL OUTPUT(LL)
   GO TO 30
200 LL = 1
   CALL OUTPUT(LL)
   GO TO 25
C - - - - -
2500 FORMAT(1H0,16X,25HITERATIONS EXCEEDED ITMAX)
9000 FORMAT(1H1,30H** BOUNDARY LAYER CALCULATIONS//)
9100 FORMAT(1H0,4HNX =,I3,5X,4HNZ =,I3,5X,3HX =,F10.5,5X,3HZ =,F10.5)
9300 FORMAT(1H0,33H** LAMINAR SEPARATION OCCURRED **)
END

```



```

SUBROUTINE INTIAL
COMMON/BLC0/ NXT,NZT,NX,NZ,NTR,NP,ITMAX,INTR,IBDY,KPHA,IPOP,N,
1      NPT,CNU,ETA,E,VGP,A(61),ETA(61),DETA(61)
COMMON/BLC1/ CA,CB,OMEGA,OMX,REDFR,X(41),Z(30),UO(30),RZ(30),
1      P1(30),P2(30),P3(41,30),UE(41,30)
COMMON/PROF/ DELV(61),F(61,41,2),U(61,41,2),V(61,41,2),B(61,41,2)
DIMENSION TITLE(20),ZC(30),YC(30)
C - - - - -
C      IBDY = 1      FLAT PLATE
C      IBDY = 2      AIRFOIL
C      ISD = 1      CALCULATED SURFACE DISTANCE
C      ISD = 2      INPUT SURFACE DISTANCE
C      IP2 = 1      P2 CALCULATED
C      IP2 = 2      P2 READ IN
C      INTR = 1      CALCULATED TRANSITION
C      INTR = 2      INPUT TRANSITION
C      KPHA = 0      DO NOT CALCULATE PHASE ANGLES
C      KPHA = 1      CALCULATE PHASE ANGLES
C      IPOP = 0      DO NOT CALCULATE PHASE COMPONENTS
C      IPOP = 1      CALCULATE PHASE COMPONENTS
C      N          NUMBER OF X-STATIONS IN ONE CYCLE
C - - - - -
      NPT = 61
      CNU = 1.6E-04
      ITMAX = 6
      READ(5,8001) TITLE
      WRITE(6,9011) TITLE
      READ(5,8000) NXT,NZT,NTR,IBDY,ISD,IP2,INTR,KPHA,IPOP,N
      READ(5,8100) ETA,E,DETA(1),VGP,UINF,CB,OMEGA,CA
      READ(5,8300) (X(I),I=1,NXT)
      WRITE(6,9200) NXT,NZT,NTR,ETA,E,DETA(1),VGP,CB,OMEGA,UINF,CA
      GO TO (10,30), ISD
10  READ(5,8400) (ZC(I),YC(I),UO(I),I=1,NZT)
      WRITE(6,9400) (I,ZC(I),YC(I),I=1,NZT)
C  CALCULATE Z
      Z(1) = 0.0
      UO(1) = UO(1)*UINF
      IF(NZT.EQ. 1) GO TO 60
      SUM1 = 0.0
      DO 20 I=2,NZT
      UO(I) = UO(I)*UINF
      SUM1 = SUM1+SQRT((ZC(I)-ZC(I-1))**2+(YC(I)-YC(I-1))**2)
20  Z(I) = SUM1
      GO TO 60
30  READ(5,8400) (Z(I),UO(I),P2(I),I=1,NZT)
C
50  IF(IP2.EQ. 2) GO TO 55
60  IF(IBDY.EQ. 1) P2(1) = 0.0
      IF(IBDY.EQ. 2) P2(1) = 1.0
55  DO 90 I=1,NZT
      IF(I.EQ. 1) GO TO 82
      IF(IP2.EQ. 2) GO TO 81
      IF(I.EQ. NZT) GO TO 70
      A1 = (Z(I)-Z(I-1))*(Z(I+1)-Z(I-1))
      A2 = (Z(I)-Z(I-1))*(Z(I+1)-Z(I))

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      A3      = (Z(I+1)-Z(I))*(Z(I+1)-Z(I-1))
      DUDS    = -(Z(I+1)-Z(I))/A1*UO(I-1)+(Z(I+1)-2.0*Z(I)+
1      Z(I-1))/A2*UO(I)+(Z(I)-Z(I-1))/A3*UO(I+1)
      GO TO 80
70  A1      = (Z(I-1)-Z(I-2))*(Z(I)-Z(I-2))
      A2      = (Z(I-1)-Z(I-2))*(Z(I)-Z(I-1))
      A3      = (Z(I)-Z(I-1))*(Z(I)-Z(I-2))
      DUDS    = (Z(I)-Z(I-1))/A1*UO(I-2)-(Z(I)-Z(I-2))/A2*UO(I-1)+
1      (2.0*Z(I)-Z(I-2)-Z(I-1))/A3*UO(I)
80  P2(I) = Z(I)/UO(I)*DUDS
81  REDFR = OMEGA*Z(I)/UO(I)
      GO TO 84
82  IF(IBDY .EQ. 1) GO TO 81
      REDFR = OMEGA/CA
84  RZ(I) = UO(I)*Z(I)/CNU
      DO 85 K=1,NXT
      OMX    = OMEGA*X(K)
      UE(K,I)=UO(I)*(1.0+CB*COS(OMX))
      IF(K .GT. 1 .AND. I .GT. 1) GO TO 83
      IF(K .EQ. 1 .AND. I .GT. 1) GO TO 73
      IF(K .GT. 1) GO TO 71
      IF(IBDY .EQ. 1) P3(K,I) = 0.0
      IF(IBDY .EQ. 2) P3(K,I) = (1.0+CB*COS(OMX))**2
      GO TO 85
71  IF(IBDY .EQ. 1) P3(K,I) = 0.0
      IF(IBDY .EQ. 2) P3(K,I) = (1.0+CB*COS(OMX))**2-REDFR*CB*SIN(OMX)
      GO TO 85
73  P3(K,I)=P2(I)*(1.0+CB*COS(OMX))**2
      GO TO 85
83  P3(K,I)=P2(I)*(UE(K,I)/UO(I))**2-CB*REDFR*SIN(OMX)
85  CONTINUE
90  CONTINUE
95  WRITE(6,9000) (Z(I),UO(I),P2(I),I=1,NZT)
100 DO 110 I=1,NZT
110 P1(I) = 0.5*(P2(I)+1.0)
      NX    = 1
      NZ    = 1

```

C

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      IF((VGP-1.0) .LE. 0.001) GO TO 105
      NP    = ALOG((ETAE/DETA(1))*(VGP-1.0)+1.0)/ALOG(VGP) + 1.0
      GO TO 112
105 NP    = ETAE/DETA(1) + 1.0
112 IF(NP .LE. NPT) GO TO 115
      WRITE(6,9300)
      STOP
115 ETA(1)= 0.0
      DO 120 J=2,NPT
      DETA(J)=VGP*DETA(J-1)
      A(J)   = 0.5*DETA(J-1)
120 ETA(J)= ETA(J-1)+DETA(J-1)
      ETANPQ= 0.25*ETA(NP)
      ETAU15= 1.5/ETA(NP)
      DO 130 J=1,NP
      ETAB   = ETA(J)/ETA(NP)
      ETAB2  = ETAB**2

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      F(J,NX,2)= ETANPQ*ETAB2*(3.0-0.5*ETAB2)
      U(J,NX,2)= 0.5*ETAB*(3.0-ETAB2)
      V(J,NX,2)= ETAU15*(1.0-ETAB2)
      B(J,NX,2)= 1.0
130  CONTINUE
      RETURN
C - - - - -
8000  FORMAT(10I3)
8001  FORMAT(20A4)
8100  FORMAT(8F10.0)
8300  FORMAT(F10.0)
8400  FORMAT(3F10.0)
9000  FORMAT(///1H0,46H** EXTERNAL STEADY STATE VELOCITY DISTRIBUTION/
1      1H ,24H** AND PRESSURE GRADIENT/1H0,4X,1HZ,9X,2HU0,8X,2HP2/
2      (1H ,3F10.5))
9011  FORMAT(1H0,20A4)
9200  FORMAT(///1H0,12H** CASE DATA/1H0,3X,6HNXT =,I3,14X,6HNZT =,
1      I3,14X,6HNTR =,I3/1H ,3X,6HETAE =,E14.6,3X,6HDETA1=,
2      E14.6,3X,6HVGP =,E14.6/1H ,3X,6HCB =,E14.6,3X,
3      6HOMEGA=,E14.6,3X,6HUNF =,E14.6/1H ,3X,6HCA =,E14.6)
9300  FORMAT(1HC,37HNP EXCEEDED NPT -- PROGRAM TERMINATED)
9400  FORMAT(//1HC,22H** INPUT BODY GEOMETRY/1HC,3H N2,6X,3HZ/C,11X,
1      3HY/C/(1H ,I3,2E14.6))
      END

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SUBROUTINE EDDY
COMMON/BLC0/ NXT,NZT,NX,NZ,NTR,NP,ITMAX,INTR,IBDY,KPHA,IPOP,N,
1      NPT,CNU,ETA,ETAE,VGP,A(61),ETA(61),DETA(61)
COMMON/BLC1/ CA,CB,OMEGA,OMX,DOFR,X(41),Z(30),UO(30),RZ(30),
1      P1(30),P2(30),P3(30),UE(41,30)
COMMON/PROF/ DELV(61),F(61,41,2),U(61,41,2),V(61,41,2),B(61,41,2)
C - - - - -
IFLG = 0
RZ2 = SQRT(RZ(NZ))
RZ216 = RZ2*0.16
RZ4 = SQRT(RZ2)
CN = 1.0
CRSQV = CN*RZ4*SQRT(ABS(V(1,NX,2)))/26.0
SUM = 0.0
F1 = 0.0
DO 30 J=2,NP
F2 = U(J,NX,2)*(1.0-U(J,NX,2))
SUM = SUM+(F1+F2)*A(J)
30 F1 = F2
RT = RZ2*SUM
IF(RT .LE. 425.) GO TO 35
IF(RT .GT. 6000.) GO TO 38
XPI = RT/425.-1.0
PI = .55*(1.0-EXP(-.243*SQRT(XPI)-2.98*XPI))
GO TO 40
35 PI = 0.0
GO TO 40
38 PI = .55
40 EDVO = .0168*(1.55/(1.0+PI))*RZ2*(U(NP,NX,2)*ETA(NP)-F(NP,NX,2))
J = 1
50 IF(IFLG .EQ. 1) GO TO 100
YOA = CRSQV*ETA(J)
EL = 1.0
IF(YOA .LT. 4.0) EL = (1.0-EXP(-YOA))**2
EDVI = RZ216*ETA(J)**2*V(J,NX,2)*EL
IF(EDVI .LT. EDVO) GO TO 200
IFLG = 1
100 EDV = EDVO
GO TO 300
200 EDV = EDVI
300 B(J,NX,2) = 1.0+EDV
J = J+1
IF(J .LE. NP) GO TO 50
RETURN
END

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SUBROUTINE ONX
COMMON/BLC0/ NXT,NZT,NX,NZ,NTR,NP,ITMAX,INTR,IBDY,KPHA,IPOP,N,
1 NPT,CNU,ETA,E,VGP,A(61),ETA(61),DETA(61)
COMMON/BLC1/ CA,CB,OMEGA,OMX,REDFR,X(41),Z(30),UO(30),RZ(30),
1 P1(30),P2(30),P3(41,30),UE(41,30)
COMMON/PROF/ DELV(61),F(61,41,2),U(61,41,2),V(61,41,2),B(61,41,2)
COMMON/BLCC/ S1(61),S2(61),S3(61),S4(61),S5(61),S6(61),
1 R1(61),R2(61),R3(61)
C - - - - -
CEL = 0.0
IF(NX .EQ. 1) GO TO 300
IF(IBDY .EQ. 1) GO TO 300
DELX = X(NX)-X(NX-1)
CEL = 2.0/(CA*DELX)
300 U(NP,NX,2) = 1.0+CB*COS(OMX)
P22 = -2.0*P2(NZ)
DO 500 J=2,NP
UB = 0.5*(U(J,NX,2)+U(J-1,NX,2))
VB = 0.5*(V(J,NX,2)+V(J-1,NX,2))
FVB = 0.5*(F(J,NX,2)*V(J,NX,2)+F(J-1,NX,2)*V(J-1,NX,2))
USB = 0.5*(U(J,NX,2)**2+U(J-1,NX,2)**2)
DERBV = (B(J,NX,2)*V(J,NX,2)-B(J-1,NX,2)*V(J-1,NX,2))/DETA(J-1)
IF(NX .GT. 1) GO TO 400
R2B = 0.0
GO TO 450
400 CUB = 0.5*(U(J,NX-1,2)+U(J-1,NX-1,2))
CFVB = 0.5*(F(J,NX-1,2)*V(J,NX-1,2)+F(J-1,NX-1,2)*V(J-1,NX-1,2))
CUSB = 0.5*(U(J,NX-1,2)**2+U(J-1,NX-1,2)**2)
CDERBV = (B(J,NX-1,2)*V(J,NX-1,2)-B(J-1,NX-1,2)*V(J-1,NX-1,2))/
1 DETA(J-1)
CR2B = CDERBV+P1(NZ)*CFVB-P2(NZ)*CUSB+P3(NX-1,NZ)
R2B = -CR2B-CEL*CUB
C
450 P1A = A(J)*P1(NZ)
S1(J) = B(J,NX,2)+P1A*F(J,NX,2)
S2(J) = -B(J-1,NX,2)+P1A*F(J-1,NX,2)
S3(J) = P1A*V(J,NX,2)
S4(J) = P1A*V(J-1,NX,2)
S5(J) = A(J)*(P22*U(J,NX,2)-CEL)
S6(J) = A(J)*(P22*U(J-1,NX,2)-CEL)
C
R1(J) = F(J-1,NX,2)-F(J,NX,2)+DETA(J-1)*UB
R3(J-1) = U(J-1,NX,2)-U(J,NX,2)+DETA(J-1)*VB
R2(J) = DETA(J-1)*(R2B-P3(NX,NZ)-(DERBV+P1(NZ)*FVB-
1 P2(NZ)*USB-CEL*UB))
500 CONTINUE
R1(1) = 0.0
R2(1) = 0.0
R3(NP) = 0.0
RETURN
END

```

```

SUBROUTINE BCONZ
COMMON/BLC0/ NXT,NZT,NX,NZ,NTR,NP,ITMAX,INTR,IBDY,KPHA,IPOP,N,
1      NPT,CNU,ETA,E,VEP,A(61),ETA(61),DETA(61)
COMMON/BLC1/ CA,CB,OMEGA,OMX,REDFR,X(41),Z(30),UO(30),RZ(30),
1      P1(30),P2(30),P3(41,30),UE(41,30)
COMMON/PROF/ DELV(61),F(61,41,2),U(61,41,2),V(61,41,2),B(61,41,2)
COMMON/BLCC/ S1(61),S2(61),S3(61),S4(61),S5(61),S6(61),
1      R1(61),R2(61),R3(61)
C - - - - -
U(NP,NX,2) = 1.0+CB*COS(OMX)
BEL      = 0.0
IF(NZ .GT. 1) BEL = 0.5*(Z(NZ)+Z(NZ-1))/(Z(NZ)-Z(NZ-1))
P1P      = P1(NZ)+BEL
P2P      = P2(NZ)+BEL
DO 100 J=2,NP
C DEFINITION OF AVERAGED QUANTITIES
FB      = 0.5*(F(J,NX,2)+F(J-1,NX,2))
UB      = 0.5*(U(J,NX,2)+U(J-1,NX,2))
VB      = 0.5*(V(J,NX,2)+V(J-1,NX,2))
FVB     = 0.5*(F(J,NX,2)*V(J,NX,2)+F(J-1,NX,2)*V(J-1,NX,2))
USB     = 0.5*(U(J,NX,2)**2+U(J-1,NX,2)**2)
DERBV   = (B(J,NX,2)*V(J,NX,2)-B(J-1,NX,2)*V(J-1,NX,2))/DETA(J-1)
IF(NZ .GT. 1) GO TO 10
CFB     = 0.0
CUB     = 0.0
CVB     = 0.0
GO TO 20
10 CFB   = 0.5*(F(J,NX,1)+F(J-1,NX,1))
CUB     = 0.5*(U(J,NX,1)+U(J-1,NX,1))
CVB     = 0.5*(V(J,NX,1)+V(J-1,NX,1))
CFVB    = 0.5*(F(J,NX,1)*V(J,NX,1)+F(J-1,NX,1)*V(J-1,NX,1))
CUSB    = 0.5*(U(J,NX,1)**2+U(J-1,NX,1)**2)
CDERBV  = (B(J,NX,1)*V(J,NX,1)-B(J-1,NX,1)*V(J-1,NX,1))/DETA(J-1)
C
C COEFFICIENTS OF THE DIFFERENCED MOMENTUM EQUATION
20 S1(J) = B(J,NX,2)/DETA(J-1)+(P1P*F(J,NX,2)-BEL*CFB)*0.5
S2(J)   = -B(J-1,NX,2)/DETA(J-1)+(P1P*F(J-1,NX,2)-BEL*CFB)*0.5
S3(J)   = 0.5*(P1P*V(J,NX,2)+BEL*CVB)
S4(J)   = 0.5*(P1P*V(J-1,NX,2)+BEL*CVB)
S5(J)   = -P2P*U(J,NX,2)
S6(J)   = -P2P*U(J-1,NX,2)
C
C DEFINITIONS OF RJ
R1(J)   = F(J-1,NX,2)-F(J,NX,2)+DETA(J-1)*UB
R3(J-1) = U(J-1,NX,2)-U(J,NX,2)+DETA(J-1)*VB
IF(NZ .EQ. 1) GO TO 30
CLB     = CDERBV+P1(NZ-1)*CFVB-P2(NZ-1)*CUSB+P3(NX,NZ-1)
CRB     = -P3(NX,NZ)+BEL*(CFVB-CUSB)-CLB
GO TO 40
30 CRB   = -P2(NZ)
40 R2(J) = CRB-(DERBV+P1P*FVB-P2P*USB-BEL*(CFB*VB-CVB*FB))
100 CONTINUE
R1(1)   = 0.0
R2(1)   = 0.0
R3(NP)  = 0.0
RETURN
END

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SUBROUTINE COEFG

```
COMMON/BLC0/ NXT,NZT,NX,NZ,NTR,NP,ITMAX,INTR,IBDY,KPHA,IPOP,N,
1 NPT,CNU,ETA,E,VGP,A(61),ETA(61),DETA(61)
COMMON/BLC1/ CA,CB,OMEGA,OMX,REDFR,X(41),Z(30),UO(30),RZ(30),
1 P1(30),P2(30),P3(41,30),UE(41,30)
COMMON/PROF/ DELV(61),F(61,41,2),U(61,41,2),V(61,41,2),B(61,41,2)
COMMON/BLCC/ S1(61),S2(61),S3(61),S4(61),S5(61),S6(61),
1 R1(61),R2(61),R3(61)
```

```
C - - - - -
U(NP,NX,2) = 1.0+CB*COS(OMX)
DELX = X(NX)-X(NX-1)
DELZ = Z(NZ)-Z(NZ-1)
ZB = 0.5*(Z(NZ)+Z(NZ-1))
UOB = 0.5*(UO(NZ)+UO(NZ-1))
CEL = ZB/DELZ
BEL = ZB/(DELX*UOB)
CEL2 = 0.5*CEL
P1B = 0.5*(P1(NZ)+P1(NZ-1))
P2B = 0.5*(P2(NZ)+P2(NZ-1))
P3B = 0.25*(P3(NX,NZ)+P3(NX-1,NZ)+P3(NX,NZ-1)+P3(NX-1,NZ-1))
P2B2 = 2.0*P2B
P3B4 = 4.0*P3B
BEL2 = 2.0*BEL
DO 500 J=2,NP
FB = 0.5*(F(J,NX,2)+F(J-1,NX,2))
FVB = 0.5*(F(J,NX,2)*V(J,NX,2)+F(J-1,NX,2)*V(J-1,NX,2))
FB4 = 0.5*(F(J,NX-1,2)+F(J-1,NX-1,2))
FVJ2 = F(J,NX,1)*V(J,NX,1)+F(J,NX-1,1)*V(J,NX-1,1)+
1 F(J,NX-1,2)*V(J,NX-1,2)
FVJ1 = F(J-1,NX,1)*V(J-1,NX,1)+F(J-1,NX-1,1)*V(J-1,NX-1,1)+
1 F(J-1,NX-1,2)*V(J-1,NX-1,2)
FBI1 = 0.25*(F(J,NX-1,1)+F(J-1,NX-1,1)+F(J,NX,1)+F(J-1,NX,1))
FVB234 = 0.5*(FVJ2+FVJ1)
UB = 0.5*(U(J,NX,2)+U(J-1,NX,2))
USB = 0.5*(U(J,NX,2)**2+U(J-1,NX,2)**2)
UB2 = 0.5*(U(J,NX,1)+U(J-1,NX,1))
UB4 = 0.5*(U(J,NX-1,2)+U(J-1,NX-1,2))
UJ1 = U(J-1,NX,1)+U(J-1,NX-1,1)+U(J-1,NX-1,2)
UJ2 = U(J,NX,1)+U(J,NX-1,1)+U(J,NX-1,2)
UBI1 = 0.25*(U(J,NX-1,1)+U(J-1,NX-1,1)+U(J,NX,1)+U(J-1,NX,1))
UBK1 = 0.25*(U(J,NX-1,1)+U(J-1,NX-1,1)+U(J,NX-1,2)+U(J-1,NX-1,2))
USJ2 = U(J,NX,1)**2+U(J,NX-1,1)**2+U(J,NX-1,2)**2
UJ1 = U(J-1,NX,1)**2+U(J-1,NX-1,1)**2+U(J-1,NX-1,2)**2
UB234 = 0.5*(UJ2+UJ1)
USB234 = 0.5*(USJ2+USJ1)
VB = 0.5*(V(J,NX,2)+V(J-1,NX,2))
VJ1 = V(J-1,NX,1)+V(J-1,NX-1,1)+V(J-1,NX-1,2)
VJ2 = V(J,NX,1)+V(J,NX-1,1)+V(J,NX-1,2)
VB234 = 0.5*(VJ2+VJ1)
BVJ1 = B(J-1,NX,1)*V(J-1,NX,1)+B(J-1,NX-1,1)*V(J-1,NX-1,1)+
1 B(J-1,NX-1,2)*V(J-1,NX-1,2)
BVJ2 = B(J,NX,1)*V(J,NX,1)+B(J,NX-1,1)*V(J,NX-1,1)+
1 B(J,NX-1,2)*V(J,NX-1,2)
DERBV = (B(J,NX,2)*V(J,NX,2)-B(J-1,NX,2)*V(J-1,NX,2))/DETA(J-1)
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C
CM1    = UB234
CM2    = UB2-2.0*UBK1
CM3    = VB234
CM4    = FB4-2.0*FBI1
CM5    = FVB234
CM6    = UB4-2.0*UBI1
CM7    = CM1*CM6-CM3*CM4
CM8    = BVJ2-BVJ1
CM9    = CM1+CM6
CM10   = -CM8+DETA(J-1)*(-P1B*CM5+USB234*P2B-P3B4+CEL2*CM7+BEL2*
1      CM2)

C
S1(J)  = B(J,NX,2)+A(J)*(P1B*F(J,NX,2)+CEL2*(FB+CM4))
S2(J)  = -B(J-1,NX,2)+A(J)*(P1B*F(J-1,NX,2)+CEL2*(FB+CM4))
S3(J)  = A(J)*(P1B*V(J,NX,2)+CEL2*(VB+CM3))
S4(J)  = A(J)*(P1B*V(J-1,NX,2)+CEL2*(VB+CM3))
S5(J)  = A(J)*(-P2B2*U(J,NX,2)-CEL2*(2.0*UB+CM9)-BEL2)
S6(J)  = A(J)*(-P2B2*U(J-1,NX,2)-CEL2*(2.0*UB+CM9)-BEL2)

C
R1(J)  = F(J-1,NX,2)-F(J,NX,2)+DETA(J-1)*UB
R2(J)  = CM10-DETA(J-1)*(DERBV+P1B*FVB-P2B*USB-CEL2*(UB*UB+CM9*
1      UB-VB*FB-CM4*VB-CM3*FB)-BEL2*UB)
R3(J-1)=U(J-1,NX,2)-U(J,NX,2)+DETA(J-1)*VB
500 CONTINUE
R3(NP)= 0.0
R1(1)  = 0.0
R2(1)  = 0.0
RETURN
END

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SUBROUTINE SOLV3
COMMON/BLC0/ NXT,NZT,NX,NZ,NTR,NP,ITMAX,INTR,IBDY,KPHA,IPOP,N,
1      NPT,CNU,ETA,E,VGP,A(61),ETA(61),DETA(61)
COMMON/PROF/ DELV(61),F(61,41,2),U(61,41,2),V(61,41,2),B(61,41,2)
COMMON/BLCC/ S1(61),S2(61),S3(61),S4(61),S5(61),S6(61),
1      R1(61),R2(61),R3(61)
DIMENSION A11(61),A12(61),A13(61),A21(61),A22(61),A23(61),
1      G11(61),G12(61),G13(61),G21(61),G22(61),G23(61),
2      W1(61),W2(61),W3(61),DELF(61),DELU(61)
C - - - - -
W1(1) = R1(1)
W2(1) = R2(1)
W3(1) = R3(1)
A11(1) = 1.0
A12(1) = 0.0
A13(1) = 0.0
A21(1) = 0.0
A22(1) = 1.0
A23(1) = 0.0
G11(2) = -1.0
G12(2) = -0.5*DETA(1)
G13(2) = 0.0
G21(2) = S4(2)
G23(2) = -2.0*S2(2)/DETA(1)
G22(2) = G23(2)+S6(2)
DO 500 J=2,NP
IF(J.EQ. 2) GO TO 100
DEN = (A13(J-1)*A21(J-1)-A23(J-1)*A11(J-1)-A(J)*
1      (A12(J-1)*A21(J-1)-A22(J-1)*A11(J-1)))
G11(J) = (A23(J-1)+A(J)*(A(J)*A21(J-1)-A22(J-1)))/DEN
G12(J) = -(1.0+G11(J)*A11(J-1))/A21(J-1)
G13(J) = (G11(J)*A13(J-1)+G12(J)*A23(J-1))/A(J)
G21(J) = (S2(J)*A21(J-1)-S4(J)*A23(J-1)+A(J)*(S4(J)*
1      A22(J-1)-S6(J)*A21(J-1)))/DEN
G22(J) = (S4(J)-G21(J)*A11(J-1))/A21(J-1)
G23(J) = (G21(J)*A12(J-1)+G22(J)*A22(J-1)-S6(J))
100 A11(J) = 1.0
A12(J) = -A(J)-G13(J)
A13(J) = A(J)*G13(J)
A21(J) = S3(J)
A22(J) = S5(J)-G23(J)
A23(J) = S1(J)+A(J)*G23(J)
W1(J) = R1(J)-G11(J)*W1(J-1)-G12(J)*W2(J-1)-G13(J)*W3(J-1)
W2(J) = R2(J)-G21(J)*W1(J-1)-G22(J)*W2(J-1)-G23(J)*W3(J-1)
W3(J) = R3(J)
500 CONTINUE
DELU(NP) = W3(NP)
E1 = W1(NP)-A12(NP)*DELU(NP)
E2 = W2(NP)-A22(NP)*DELU(NP)
DELV(NP) = (E2*A11(NP)-E1*A21(NP))/(A23(NP)*A11(NP)-A13(NP)*
1      A21(NP))
DELF(NP) = (E1-A13(NP)*DELV(NP))/A11(NP)

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      J      = NP
600 J      = J-1
      E3     = W3(J)-DELU(J+1)+A(J+1)*DELV(J+1)
      DELV(J) = (A11(J)*(W2(J)+E3*A22(J))-A21(J)*W1(J)-E3*A21(J)*A12(J)
1          )/(A21(J)*A12(J)*A(J+1)-A21(J)*A13(J)-A(J+1)*
2          A22(J)*A11(J)+A23(J)*A11(J))
      DELU(J) = -A(J+1)*DELV(J)-E3
      DELF(J) = (W1(J)-A12(J)*DELU(J)-A13(J)*DELV(J))/A11(J)
      IF(J.GT. 1) GO TO 600
      WRITE(6,9100) V(1,NX,2),DELV(1)
      DO 700 J=1,NP
      F(J,NX,2)= F(J,NX,2)+DELF(J)
      U(J,NX,2)= U(J,NX,2)+DELU(J)
700 V(J,NX,2)= V(J,NX,2)+DELV(J)
      U(1,NX,2)= 0.0
      RETURN
C - - - - -
9100 FORMAT(1H ,5X,8HV(WALL)=,E14.6,5X,6HDELVW=,E14.6)
      END

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SUBROUTINE OUTPUT(LL)
COMMON/BLC0/ NXT,NZT,NX,NZ,NTR,NP,ITMAX,INTR,IBDY,KPHA,IPOP,N,
1      NPT,CNU,ETA,E,VEP,A(61),ETA(61),DETA(61)
COMMON/BLC1/ CA,CB,OMEGA,OMX,REDFR,X(41),Z(30),UO(30),RZ(30),
1      P1(30),P2(30),P3(41,30),UE(41,30)
COMMON/PROF/ DELV(61),F(61,41,2),U(61,41,2),V(61,41,2),B(61,41,2)
DIMENSION NPK(41),RHT(30),RTHETA(30),ALFAA(61,41),ALFAB(61,41),
1      SUMA(61),SUMB(61)
C - - - - -
IF(LL .EQ. 2) GO TO 150
NPK(NX)=NP
IF(RZ(NZ) .EQ. 0.0) GO TO 140
ZRZ = Z(NZ)/SQRT(RZ(NZ))
DELSTR= ZRZ*(ETA(NP)-F(NP,NX,2)/U(NP,NX,2))
CF = 2.0*V(1,NX,2)/SQRT(RZ(NZ))
RDELST= UO(NZ)*DELSTR/CNU
SUM1 = 0.0
F1 = U(1,NX,2)/U(NP,NX,2)*(1.0-U(1,NX,2)/U(NP,NX,2))
DO 50 J=2,NP
F2 = U(J,NX,2)/U(NP,NX,2)*(1.0-U(J,NX,2)/U(NP,NX,2))
SUM1 = SUM1+(F1+F2)*A(J)
50 F1 = F2
THETA = ZRZ*SUM1
RHTA = UO(NZ)*THETA/CNU
H = DELSTR/THETA
C
C CHECK FOR TRANSITION IF IT IS TO BE CALCULATED
IF(INTR .EQ. 2) GO TO 150
IF(P2(NZ) .GE. 0.0) GO TO 150
IF(NZ .GE. NTR) GO TO 150
IF(NX .GT. 1) GO TO 150
IF(NZ .EQ. 1) GO TO 150
IF(NZ .EQ. NZT) GO TO 150
RZTR = RZ(NZ)
RTHETA(NZ)=RHTA
RHT(NZ) = 1.174*(1.+22400./RZTR)*RZTR**0.46
WRITE(6,9500) RTHETA(NZ),RHT(NZ)
IF(RTHETA(NZ)-RHT(NZ)) 150,110,120
110 ZTR = Z(NZ)
WRITE(6,9600) ZTR
GO TO 150
120 ZTR1 = Z(NZ-1)
ZTR2 = Z(NZ)
DRTH1 = RHT(NZ-1)-RTHETA(NZ-1)
DRTH2 = RHT(NZ)-RTHETA(NZ)
ZTR = ZTR1+(DRTH1*(ZTR2-ZTR1))/(DRTH1-DRTH2)
UOTR = UO(NZ-1)+((ZTR-ZTR1)/(ZTR2-ZTR1))*(UO(NZ)-UO(NZ-1))
P2TR = P2(NZ-1)+((ZTR-ZTR1)/(ZTR2-ZTR1))*(P2(NZ)-P2(NZ-1))
I = NZT+2
IF(NZT .EQ. 30) I = NZT+1
100 I = I-1
Z(I) = Z(I-1)
RZ(I) = RZ(I-1)
UO(I) = UO(I-1)
P1(I) = P1(I-1)

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P2(I) = P2(I-1)
DO 90 K=1,NXT
UE(K,I)=UE(K,I-1)
90 P3(K,I)=P3(K,I-1)
IF(I .GT. (NZ+1)) GO TO 100
P2(NZ) = P2TR
P1(NZ) = 0.5*(1.0+P2(NZ))
Z(NZ) = ZTR
UO(NZ) = UOTR
RZ(NZ) = Z(NZ)*UO(NZ)/CNU
UE(1,NZ) = UO(NZ)*(1.0+CB*COS(OMEGA*X(1)))
P3(1,NZ) = P2(NZ)*(1.0+CB*COS(OMEGA*X(1)))**2
DO 125 K=2,NXT
UE(K,NZ) = UO(NZ)*(1.0+CB*COS(OMEGA*X(K)))
REDFR = OMEGA*Z(NZ)/UO(NZ)
P3(K,NZ) = P2(NZ)*(UE(K,NZ)/UO(NZ))**2-CB*REDFR*SIN(OMEGA*X(K))
125 CONTINUE
NTR = NZ
IF(NZT .LT. 30) NZT = NZT+1
DO 130 J=1,NP
F(J,NX,2) = F(J,NX,1)
U(J,NX,2) = U(J,NX,1)
V(J,NX,2) = V(J,NX,1)
130 B(J,NX,2) = B(J,NX,1)
WRITE(6,9900) P2TR,UOTR,ZTR
IF(NTR .EQ. NZ) RETURN
140 RTHTA = 0.0
RTHETA(NZ)=0.0
C
150 NPO = NP
NP1 = NP+1
IF(LL .EQ. 2) NP = NP+2
IF(NP .GT. NPT) NP = NPT
DO 160 J=NP1,NPT
F(J,NX,2) = U(NPO,NX,2)*(ETA(J)-ETA(NPO))+F(NPO,NX,2)
U(J,NX,2) = U(NPO,NX,2)
V(J,NX,2) = V(NPO,NX,2)
160 B(J,NX,2) = B(NPO,NX,2)
IF(LL .EQ. 2) RETURN
C
180 WRITE(6,9010)
NPM1 = NP-1
WRITE(6,9000) (J,ETA(J),F(J,NX,2),U(J,NX,2),V(J,NX,2),B(J,NX,2),
1 J=1,NPM1,3)
WRITE(6,9000) NP,ETA(NP),F(NP,NX,2),U(NP,NX,2),V(NP,NX,2),
1 B(NP,NX,2)
IF(NZ .EQ. 1) GO TO 10
WRITE(6,9200) DELSTR,THETA,CF,RDELST,RTHTA,RZ(NZ),H,REDFR
IF(NXT .EQ. 1) GO TO 10
DELX = X(2)-X(1)
IF(IPOP .EQ. 0) GO TO 196
IF(NZ .LT. NTR) GO TO 195
C CALCULATE IN-PHASE AND OUT-OF-PHASE COMPONENTS OF AN OSCILLATING
C TURBULENT FLOW
COMX = COS(OMX)
SOMX = SIN(OMX)

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DO 190 J=1,NPT
ALFAA(J,NX)= U(J,NX,2)*COMX
190 ALFAB(J,NX)= U(J,NX,2)*SOMX
IF(NX .LT. NXT) GO TO 10
I1      = 2
I2      = N-1
191 COEFF = 2.0/(CB*FLOAT(N-1))
DO 195 J=1,NP
SUMA(J)=0.0
SUMB(J)=0.0
DO 198 I=I1,I2
SUMA(J)=SUMA(J)+ALFAA(J,I)
198 SUMB(J)=SUMB(J)+ALFAB(J,I)
SUMA(J)=(0.5*(ALFAA(J,I1-1)+ALFAA(J,I2+1))+SUMA(J))*COEFF
195 SUMB(J)=-(0.5*(ALFAB(J,I1-1)+ALFAB(J,I2+1))+SUMB(J))*COEFF
WRITE(6,9300) X(I2+1), (J,SUMA(J),SUMB(J),J=1,NP)
IF(I2 .EQ. (NXT-1)) GO TO 196
I1      = I1+(N-1)
I2      = I2+(N-1)
IF(I2 .LE. (NXT-1)) GO TO 191
WRITE(6,9700)
C
196 IF(KPHA .EQ. 0) GO TO 10
IF(NX .LT. NXT) GO TO 10
C CALCULATE PHASE ANGLES
I1      = 2
I2      = N-1
211 AV1   = 0.0
ADLSTR= 0.0
DO 210 I=I1,I2
ADLSTR= ADLSTR+(ZRZ*(ETA(NP)-F(NP,I,2)/U(NP,I,2)))
210 AV1   = AV1+V(1,I,2)
D1      = ZRZ*(ETA(NP)-F(NP,I1-1,2)/U(NP,I1-1,2))
D2      = ZRZ*(ETA(NP)-F(NP,I2+1,2)/U(NP,I2+1,2))
ADLSTR= OMEGA/6.2832*(0.5*(D1+D2)+ADLSTR)*DELX
AV1     = OMEGA/6.2832*(0.5*(V(1,I1-1,2)+V(1,I2+1,2))+AV1)*DELX
ALF2    = 0.0
BTA2    = 0.0
ALFBTA= 0.0
ALFASQ= 0.0
BETASQ= 0.0
DO 220 I=I1,I2
DLS     = ZRZ*(ETA(NP)-F(NP,I,2)/U(NP,I,2))
ALF2    = ALF2+(UE(I,NZ)-UO(NZ))*(DLS-ADLSTR)
BTA2    = BTA2+(DLS-ADLSTR)**2
ALFBTA= ALFBTA+(UE(I,NZ)-UO(NZ))*(V(1,I,2)-AV1)
ALFASQ= ALFASQ+(UE(I,NZ)-UO(NZ))**2
220 BETASQ= BETASQ+(V(1,I,2)-AV1)**2
C CALCULATE PHASE ANGLE BETWEEN WALL SHEAR AND UE
ALFBTA= (0.5*((UE(I1-1,NZ)-UO(NZ))*(V(1,I1-1,2)-AV1)+
1 (UE(I2+1,NZ)-UO(NZ))*(V(1,I2+1,2)-AV1))+ALFBTA)*DELX
ALFASQ= (0.5*((UE(I1-1,NZ)-UO(NZ))**2+(UE(I2+1,NZ)-UO(NZ))**2)+
1 ALFASQ)*DELX
BETASQ= (0.5*((V(1,I1-1,2)-AV1)**2+(V(1,I2+1,2)-AV1)**2)+
1 BETASQ)*DELX
PHI     = ARCOS(ALFBTA/SQRT(ALFASQ*BETASQ))*57.29578

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C  CALCULATE PHASE ANGLE BETWEEN DISPLACEMENT THICKNESS AND UE
    ALF2 = (0.5*((UE(I1-1,NZ)-UO(NZ))*(D1-ADLSTR)+(UE(I2+1,NZ)-
1      UO(NZ))*(D2-ADLSTR))+ALF2)*DELX
    BTA2 = (0.5*((D1-ADLSTR)**2+(D2-ADLSTR)**2)+BTA2)*DELX
    PHI2 = ARCOS(ALF2/SQRT(ALFASQ*BTA2))*57.29578
    WRITE(6,9400) X(I2+1),PHI,PHI2
C
    IF(I2 .EQ. (NXT-1)) GO TO 10
    I1 = I1+(N-1)
    I2 = I2+(N-1)
    IF(I2 .LE. (NXT-1)) GO TO 211
    WRITE(6,9700)
C
C
10  IF(NX .EQ. NXT) GO TO 200
    NX = NX+1
300 IF(NZ .GT. 1) GO TO 350
C  INITIAL GUESS IN NX DIRECTION (NZ=1)
310 DO 400 J=1,NPT
    F(J,NX,2)= F(J,NX-1,2)
    U(J,NX,2)= U(J,NX-1,2)
    V(J,NX,2)= V(J,NX-1,2)
    B(J,NX,2)= B(J,NX-1,2)
400 CONTINUE
    GO TO 370
350 IF(NX .EQ. 1) GO TO 500
C  INITIAL GUESS IN NX DIRECTION (NZ .GT. 1)
    GO TO 310
370 IF(NZ .EQ. 1) RETURN
    NP = NPK(NX)
    IF(NX .EQ. 1) RETURN
    IF(NP .LT. NPK(NX-1)) NP = NPK(NX-1)
    RETURN
200 NX = 1
    WRITE(6,9800)
    IF(NZ .EQ. NZT) STOP
    NZ = NZ+1
C  SHIFT ALL NX PROFILES IN THE NZ DIRECTION
500 DO 550 K=1,NXT
    DO 520 J=1,NPT
        F(J,K,1)= F(J,K,2)
        U(J,K,1)= U(J,K,2)
        V(J,K,1)= V(J,K,2)
        B(J,K,1)= B(J,K,2)
520 CONTINUE
550 CONTINUE
    GO TO 370
C  - - - - -
9010 FORMAT(1H0,2X,1HJ,4X,3HETA,10X,1HF,13X,1HU,13X,1HV,13X,1HB)
9000 FORMAT(1H ,I3,F10.6,4E14.6)
9200 FORMAT(1H0,7HDELSTR=,E14.6,3X,7HTHETA =,E14.6,3X,7HCF =,
1      E14.6/1H ,7HRDELST=,E14.6,3X,7HRTHTA =,E14.6,3X,7HRZ =,
2      E14.6/1H ,7HH =,E14.6,3X,7HREDFR =,E14.6)
9300 FORMAT(/1H0,4X,22H** PHASE COMPONENTS **/1H ,18HCYCLE ENDS WITH X=
1,      E14.6/1H0,2X,1HJ,3X,8HIN-PHASE,4X,12HOUT-OF-PHASE/
2      (1H ,I3,2E14.6))

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9400 FORMAT(1H0,18HCYCLE ENDS WITH X=,E14.6/
1      1H ,38HPHASE ANGLE BETWEEN WALL SHEAR AND UE=,12X,E14.6/
2      1H ,50MPHASE ANGLE BETWEEN DISPLACEMENT THICKNESS AND UE=,
3      E14.6)
9500 FORMAT(1H0,7HRTHTA=,E14.6,3X,5HRTHT=,E14.6)
9600 FORMAT(1H0,28H** TRANSITION OCCURRED AT Z=,E14.6)
9700 FORMAT(1H0,67H** INPUT NXT DOES NOT CONTAIN EQUAL INTERVALS OF PHA
1SE ANGLE PERIOD)
9800 FORMAT(1H0,39(2H*-)//)
9900 FORMAT(1H0,3HP2=,E14.6,3X,3HUO=,E14.6,3X,2HZ=,E14.6)
END

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*** TEST CASE 1 - HOWARTH'S LAMINAR FLOW

** CASE DATA

NYT = 1	NRT = 21	NTR = 99
ETA = 0.800000E+01	DETA1 = 0.200000E+00	VGP = 0.100000E+01
CB = 0	OMEGA = 0.100000E+01	UINF = 0.100000E+01
CA = 0.200000E+03		

** EXTERNAL STEADY STATE VELOCITY DISTRIBUTION

** AND PRESSURE GRADIENT

Z	UO	P2
0.0	1.00000	0.0
0.05000	0.99375	-0.00629
0.10000	0.98750	-0.01256
0.15000	0.98125	-0.01911
0.20000	0.97500	-0.02564
0.25000	0.96875	-0.03226
0.30000	0.96250	-0.03896
0.35000	0.95625	-0.04575
0.40000	0.95000	-0.05263
0.45000	0.94375	-0.05960
0.50000	0.93750	-0.06667
0.55000	0.93125	-0.07369
0.60000	0.92500	-0.08102
0.65000	0.91875	-0.08825
0.70000	0.91250	-0.09579
0.75000	0.90625	-0.10345
0.80000	0.90000	-0.11092
0.85000	0.89375	-0.11880
0.90000	0.88750	-0.12651
0.95000	0.88125	-0.13461
1.00000	0.87500	-0.14294

** BOUNDARY LAYER CALCULATIONS

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NX = 1      NZ = 1      X = 0.0      Z = 0.0
V(WALL)= 0.187500E+00    DELVW= 0.198620E+00
V(WALL)= 0.386120E+00    DELVW= -0.486990E-01
V(WALL)= 0.337421E+00    DELVW= -0.536625E-02
V(WALL)= 0.332055E+00    DELVW= -0.669347E-04

J      ETA      F      U      V      B
1  0.0      0.0      0.0      0.331988E+00  0.100000E+01
4  0.600000  0.597017E-01  0.198830E+00  0.329903E+00  0.100000E+01
7  1.200000  0.237744E+00  0.393448E+00  0.316348E+00  0.100000E+01
10 1.799999  0.528925E+00  0.574169E+00  0.282737E+00  0.100000E+01
13 2.399999  0.921103E+00  0.728224E+00  0.228076E+00  0.100000E+01
16 2.999998  0.139494E+01  0.845316E+00  0.161549E+00  0.100000E+01
19 3.599998  0.192707E+01  0.922807E+00  0.983551E-01  0.100000E+01
22 4.199997  0.249519E+01  0.966684E+00  0.507159E-01  0.100000E+01
25 4.799996  0.308226E+01  0.987694E+00  0.219445E-01  0.100000E+01
28 5.399996  0.367779E+01  0.996142E+00  0.792393E-02  0.100000E+01
31 5.999995  0.427648E+01  0.998981E+00  0.237917E-02  0.100000E+01
34 6.599995  0.487618E+01  0.999776E+00  0.592262E-03  0.100000E+01
37 7.199994  0.547611E+01  0.999961E+00  0.121874E-03  0.100000E+01
40 7.799994  0.607631E+01  0.999997E+00  0.206583E-04  0.100000E+01
41 7.999993  0.627636E+01  0.100000E+01  0.109428E-04  0.100000E+01
*-*-*-*-*

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NX = 1      NZ = 2      X = 0.0      Z = 0.05000
V(WALL)= 0.331988E+00    DELVW= -0.972310E-02
V(WALL)= 0.322265E+00    DELVW= -0.450226E-04

J      ETA      F      U      V      B
1  0.0      0.0      0.0      0.322220E+00  0.100000E+01
4  0.600000  0.581813E-01  0.194094E+00  0.323867E+00  0.100000E+01
7  1.200000  0.232522E+00  0.386156E+00  0.313812E+00  0.100000E+01
10 1.799999  0.519068E+00  0.566277E+00  0.283169E+00  0.100000E+01
13 2.399999  0.906733E+00  0.721254E+00  0.230550E+00  0.100000E+01
16 2.999998  0.137691E+01  0.840133E+00  0.164844E+00  0.100000E+01
19 3.599998  0.190651E+01  0.919551E+00  0.101347E+00  0.100000E+01
22 4.199997  0.247316E+01  0.964962E+00  0.527956E-01  0.100000E+01
25 4.799996  0.305951E+01  0.986931E+00  0.230901E-01  0.100000E+01
28 5.399996  0.365474E+01  0.995860E+00  0.843160E-02  0.100000E+01
31 5.999995  0.425332E+01  0.998894E+00  0.256168E-02  0.100000E+01
34 6.599995  0.485297E+01  0.999754E+00  0.645788E-03  0.100000E+01
37 7.199994  0.545289E+01  0.999957E+00  0.134743E-03  0.100000E+01
40 7.799994  0.605288E+01  0.999996E+00  0.232829E-04  0.100000E+01
41 7.999993  0.625288E+01  0.100000E+01  0.124532E-04  0.100000E+01
DELSTR= 0.495709E-02    THETA = 0.189808E-02    CF = 0.365695E-01
RDELST= 0.307882E+02    RTHIA = 0.117888E+02    RZ = 0.310547E+03
H = 0.261164E+01    REDFR = 0.114286E+01
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NX = 1      NZ = 3      X = 0.0      Z = 0.10000
V(WALL)= 0.322220E+00    DELVW= -0.998403E-C2
V(WALL)= 0.312236E+00    DELVW= -0.657526E-C4
J      ETA      F      U      V      B
1      0.0      0.0      0.0      0.312170E+00    0.100000E+01
4      0.600000    0.566133E-01    0.189204E+00    0.317602E+00    0.100000E+01
7      1.200000    0.227123E+00    0.378592E+00    0.311117E+00    0.100000E+01
10     1.799999    0.508849E+00    0.558047E+00    0.283535E+00    0.100000E+01
13     2.399999    0.891792E+00    0.713937E+00    0.233062E+00    0.100000E+01
16     2.999998    0.135810E+01    0.834651E+00    0.168257E+00    0.100000E+01
19     3.599998    0.188502E+01    0.916079E+00    0.104488E+00    0.100000E+01
22     4.199997    0.245010E+01    0.963110E+00    0.550052E-01    0.100000E+01
25     4.799996    0.303568E+01    0.986103E+00    0.243212E-01    0.100000E+01
28     5.399996    0.363058E+01    0.995550E+00    0.898349E-02    0.100000E+01
31     5.999995    0.422905E+01    0.998798E+00    0.276242E-02    0.100000E+01
34     6.599995    0.482866E+01    0.999729E+00    0.705297E-03    0.100000E+01
37     7.199994    0.542858E+01    0.999952E+00    0.149013E-03    0.100000E+01
40     7.799994    0.602857E+01    0.999996E+00    0.261163E-04    0.100000E+01
41     7.999993    0.622857E+01    0.100000E+01    0.141165E-04    0.100000E+01
DELSTR= 0.713040E-02    THETA = 0.271337E-02    CF = 0.251312E-01
RDELST= 0.440079E+02    RTHETA = 0.167466E+02    RZ = 0.617137E+03
H      = 0.262788E+01    REDFR = 0.114286E+01
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NX = 1      NZ = 4      X = 0.0      Z = 0.15000
V(WALL)= 0.312170E+00    DELVW= -0.102826E-01
V(WALL)= 0.301887E+00    DELVW= -0.817980E-04
J      ETA      F      U      V      B
1      0.0      0.0      0.0      0.301805E+00    0.100000E+01
4      0.600000    0.549919E-01    0.184141E+00    0.311074E+00    0.100000E+01
7      1.200000    0.221523E+00    0.370720E+00    0.308233E+00    0.100000E+01
10     1.799999    0.498217E+00    0.549423E+00    0.283818E+00    0.100000E+01
13     2.399999    0.876195E+00    0.706213E+00    0.235609E+00    0.100000E+01
16     2.999998    0.133841E+01    0.828816E+00    0.171804E+00    0.100000E+01
19     3.599998    0.186247E+01    0.912348E+00    0.107804E+00    0.100000E+01
22     4.199997    0.242585E+01    0.961100E+00    0.573694E-01    0.100000E+01
25     4.799996    0.301058E+01    0.985194E+00    0.256553E-01    0.100000E+01
28     5.399996    0.360513E+01    0.995206E+00    0.958888E-02    0.100000E+01
31     5.999995    0.420347E+01    0.998690E+00    0.298533E-02    0.100000E+01
34     6.599995    0.480205E+01    0.999701E+00    0.772255E-03    0.100000E+01
37     7.199994    0.540296E+01    0.999946E+00    0.165874E-03    0.100000E+01
40     7.799994    0.600294E+01    0.999995E+00    0.298419E-04    0.100000E+01
41     7.999993    0.620294E+01    0.100000E+01    0.162542E-04    0.100000E+01
DELSTR= 0.888743E-02    THETA = 0.335972E-02    CF = 0.199013E-01
RDELST= 0.545049E+02    RTHETA = 0.206045E+02    RZ = 0.919922E+03
H      = 0.264529E+01    REDFR = 0.114286E+01
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NX = 1      NZ = 5      X = 0.0      Z = 0.20000
V(WALL)= 0.301805E+00    DELVW= -0.106050E-01
V(WALL)= 0.291200E+00    DELVW= -0.978548E-04
J      ETA      F      U      V      B
1  0.0      0.0      0.0      0.291103E+00  0.100000E+01
4  0.600000  0.533129E-01  0.178890E+00  0.304263E+00  0.100000E+01
7  1.200000  0.215707E+00  0.362511E+00  0.305143E+00  0.100000E+01
10 1.799999  0.487134E+00  0.540372E+00  0.284004E+00  0.100000E+01
13 2.399999  0.859882E+00  0.698041E+00  0.238190E+00  0.100000E+01
16 2.999998  0.131775E+01  0.822587E+00  0.175493E+00  0.100000E+01
19 3.599998  0.183873E+01  0.908327E+00  0.111310E+00  0.100000E+01
22 4.199997  0.240028E+01  0.958911E+00  0.599049E-01  0.100000E+01
25 4.799996  0.298408E+01  0.984194E+00  0.271057E-01  0.100000E+01
28 5.399996  0.357824E+01  0.994824E+00  0.102561E-01  0.100000E+01
31 5.999995  0.417644E+01  0.998568E+00  0.323399E-02  0.100000E+01
34 6.599995  0.477597E+01  0.999669E+00  0.848130E-03  0.100000E+01
37 7.199994  0.537587E+01  0.999939E+00  0.184847E-03  0.100000E+01
40 7.799994  0.597586E+01  0.999995E+00  0.337430E-04  0.100000E+01
41 7.999993  0.617586E+01  0.100000E+01  0.186796E-04  0.100000E+01
DELSTR= 0.104503E-01  THETA = 0.392278E-02  CF = 0.166770E-01
RDELST= 0.636816E+02  RTHETA = 0.239044E+02  RZ = 0.121875E+04
H = 0.266401E+01  REDFR = 0.114286E+01
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NX = 1      NZ = 6      X = 0.0      Z = 0.25000
V(WALL)= 0.291103E+00    DELVW= -0.109539E-01
V(WALL)= 0.280149E+00    DELVW= -0.115092E-03
J      ETA      F      U      V      B
1  0.0      0.0      0.0      0.280033E+00  0.100000E+01
4  0.600000  0.515713E-01  0.173435E+00  0.297141E+00  0.100000E+01
7  1.200000  0.209655E+00  0.353935E+00  0.301824E+00  0.100000E+01
10 1.799999  0.475562E+00  0.530849E+00  0.284080E+00  0.100000E+01
13 2.399999  0.842785E+00  0.689375E+00  0.240801E+00  0.100000E+01
16 2.999998  0.129602E+01  0.815920E+00  0.179335E+00  0.100000E+01
19 3.599998  0.181370E+01  0.903981E+00  0.115025E+00  0.100000E+01
22 4.199997  0.237325E+01  0.956519E+00  0.626322E-01  0.100000E+01
25 4.799996  0.295604E+01  0.983088E+00  0.286873E-01  0.100000E+01
28 5.399996  0.354976E+01  0.994396E+00  0.109933E-01  0.100000E+01
31 5.999995  0.414780E+01  0.998430E+00  0.351333E-02  0.100000E+01
34 6.599995  0.474729E+01  0.999632E+00  0.934214E-03  0.100000E+01
37 7.199994  0.534718E+01  0.999932E+00  0.206550E-03  0.100000E+01
40 7.799994  0.594716E+01  0.999994E+00  0.387246E-04  0.100000E+01
41 7.999993  0.614716E+01  0.100000E+01  0.214701E-04  0.100000E+01
DELSTR= 0.119059E-01  THETA = 0.443553E-02  CF = 0.143954E-01
RDELST= 0.720863E+02  RTHETA = 0.268557E+02  RZ = 0.151367E+04
H = 0.268420E+01  REDFR = 0.114286E+01
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NX = 1      NZ = 7      X = 0.0      Z = 0.30000
V(WALL)= 0.280033E+00    DELVW= -0.113329E-01
V(WALL)= 0.268701E+00    DELVW= -0.133248E-03
J      ETA      F      U      V      B
1 0.0 0.0 0.0 0.268567E+00 0.100000E+01
4 0.600000 0.497615E-01 0.167759E+00 0.289678E+00 0.100000E+01
7 1.200000 0.203345E+00 0.1956E+00 0.298249E+00 0.100000E+01
10 1.799999 0.463452E+00 0.520807E+00 0.284027E+00 0.100000E+01
13 2.399999 0.824825E+00 0.680158E+00 0.243439E+00 0.100000E+01
16 2.999998 0.127311E+01 0.808762E+00 0.183340E+00 0.100000E+01
19 3.599998 0.178724E+01 0.899264E+00 0.118970E+00 0.100000E+01
22 4.199997 0.234461E+01 0.953895E+00 0.655742E-01 0.100000E+01
25 4.799996 0.292627E+01 0.981861E+00 0.304194E-01 0.100000E+01
28 5.399996 0.351950E+01 0.993916E+00 0.118126E-01 0.100000E+01
31 5.999995 0.411737E+01 0.998273E+00 0.382744E-02 0.100000E+01
34 6.599995 0.471680E+01 0.999590E+00 0.103287E-02 0.100000E+01
37 7.199994 0.531667E+01 0.999922E+00 0.232568E-03 0.100000E+01
40 7.799994 0.591665E+01 0.999993E+00 0.444706E-04 0.100000E+01
41 7.999993 0.611665E+01 0.100000E+01 0.252692E-04 0.100000E+01
DELSTR= 0.132999E-01 THETA = 0.491485E-02 CF = 0.126439E-01
RDELST= 0.800073E+02 RTHTA = 0.295659E+02 RZ = 0.180469E+04
H = 0.270607E+01 REDFR = 0.114286E+01
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NX = 1      NZ = 8      X = 0.0      Z = 0.35000
V(WALL)= 0.268567E+00    DELVW= -0.117447E-01
V(WALL)= 0.256823E+00    DELVW= -0.155162E-03
J      ETA      F      U      V      B
1 0.0 0.0 0.0 0.256667E+00 0.100000E+01
4 0.600000 0.478772E-01 0.161839E+00 0.281840E+00 0.100000E+01
7 1.200000 0.196752E+00 0.335533E+00 0.294388E+00 0.100000E+01
10 1.799999 0.450749E+00 0.510189E+00 0.283825E+00 0.100000E+01
13 2.399999 0.805910E+00 0.670325E+00 0.246099E+00 0.100000E+01
16 2.999998 0.124889E+01 0.801049E+00 0.187519E+00 0.100000E+01
19 3.599998 0.175916E+01 0.894127E+00 0.123172E+00 0.100000E+01
22 4.199997 0.231415E+01 0.951002E+00 0.687604E-01 0.100000E+01
25 4.799996 0.289457E+01 0.980491E+00 0.323235E-01 0.100000E+01
28 5.399996 0.348725E+01 0.993372E+00 0.127264E-01 0.100000E+01
31 5.999995 0.408492E+01 0.998093E+00 0.418363E-02 0.100000E+01
34 6.599995 0.468429E+01 0.999540E+00 0.114610E-02 0.100000E+01
37 7.199994 0.528414E+01 0.999912E+00 0.262093E-03 0.100000E+01
40 7.799994 0.588412E+01 0.999992E+00 0.514313E-04 0.100000E+01
41 7.999993 0.608412E+01 0.100000E+01 0.297837E-04 0.100000E+01
DELSTR= 0.146614E-01 THETA = 0.537079E-02 CF = 0.112238E-01
RDELST= 0.876247E+02 RTHTA = 0.320989E+02 RZ = 0.209180E+04
H = 0.272984E+01 REDFR = 0.114286E+01
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NX = 1      NZ = 9      X = 0.0      Z = 0.40000
V(WALL)= 0.256667E+00    DELVW= -0.121982E-01
V(WALL)= 0.244469E+00    DELVW= -0.181634E-03
J      ETA      F      U      V      B
1  0.0      0.0      0.0      0.244287E+00  0.100000E+01
4  0.600000  0.459102E-01  0.155649E+00  0.273584E+00  0.100000E+01
7  1.200000  0.189844E+00  0.325616E+00  0.290204E+00  0.100000E+01
10 1.799999  0.437384E+00  0.498923E+00  0.283448E+00  0.100000E+01
13 2.399999  0.785926E+00  0.659796E+00  0.248771E+00  0.100000E+01
16 2.999998  0.122320E+01  0.792702E+00  0.191889E+00  0.100000E+01
19 3.599998  0.172927E+01  0.888503E+00  0.127658E+00  0.100000E+01
22 4.199997  0.228164E+01  0.947797E+00  0.722225E-01  0.100000E+01
25 4.799996  0.286068E+01  0.978954E+00  0.344275E-01  0.100000E+01
28 5.399996  0.345274E+01  0.992754E+00  0.137529E-01  0.100000E+01
31 5.999995  0.405017E+01  0.997885E+00  0.458946E-02  0.100000E+01
34 6.599995  0.464947E+01  0.999483E+00  0.127798E-02  0.100000E+01
37 7.199994  0.524931E+01  0.999899E+00  0.297499E-03  0.100000E+01
40 7.799994  0.584928E+01  0.999991E+00  0.596816E-04  0.100000E+01
41 7.999993  0.604928E+01  0.100000E+01  0.350289E-04  0.100000E+01
DELSTR= 0.160111E-01  THETA = 0.580995E-02  CF = 0.100253E-01
RDELST= 0.950658E+02  RTHTA = 0.344966E+02  RZ = 0.237500E+04
H = 0.275581E+01  REDFR = 0.114286E+01
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NX = 1      NZ = 10     X = 0.0      Z = 0.45000
V(WALL)= 0.244287E+00    DELVW= -0.126964E-01
V(WALL)= 0.231591E+00    DELVW= -0.212776E-03
J      ETA      F      U      V      B
1  0.0      0.0      0.0      0.231378E+00  0.100000E+01
4  0.600000  0.438514E-01  0.149158E+00  0.264859E+00  0.100000E+01
7  1.200000  0.182585E+00  0.315144E+00  0.285651E+00  0.100000E+01
10 1.799999  0.423279E+00  0.486928E+00  0.282863E+00  0.100000E+01
13 2.399999  0.764739E+00  0.648475E+00  0.251448E+00  0.100000E+01
16 2.999998  0.119584E+01  0.783629E+00  0.196463E+00  0.100000E+01
19 3.599998  0.169733E+01  0.882315E+00  0.132466E+00  0.100000E+01
22 4.199997  0.224680E+01  0.944224E+00  0.760038E-01  0.100000E+01
25 4.799996  0.282428E+01  0.977216E+00  0.367653E-01  0.100000E+01
28 5.399996  0.341564E+01  0.992045E+00  0.149122E-01  0.100000E+01
31 5.999995  0.401281E+01  0.997643E+00  0.505615E-02  0.100000E+01
34 6.599995  0.461202E+01  0.999414E+00  0.143137E-02  0.100000E+01
37 7.199994  0.521184E+01  0.999883E+00  0.338957E-03  0.100000E+01
40 7.799994  0.581181E+01  0.999989E+00  0.702648E-04  0.100000E+01
41 7.999993  0.601181E+01  0.100000E+01  0.411659E-04  0.100000E+01
DELSTR= 0.173658E-01  THETA = 0.623697E-02  CF = 0.898210E-02
RDELST= 0.102431E+03  RTHTA = 0.367883E+02  RZ = 0.265430E+04
H = 0.278434E+01  REDFR = 0.114286E+01
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1 Report No NASA TM-78470	2 Government Accession No.	3 Recipient's Catalog No	
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7 Author(s) Tuncer Cebeci* and Lawrence W. Carr		8 Performing Organization Report No. A-7340	
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15 Supplementary Notes *California State University, 1250 N Bellflower Blvd., Long Beach, Calif. 90840			
16 Abstract <p>There are many unsteady boundary-layer problems in which it is necessary to account for fluctuations in the external flow. These fluctuations may change in direction and magnitude and, in most turbulent flow cases, may be regarded as low frequency fluctuations superimposed on the turbulence energy spectrum. The computer program described here provides solutions of two-dimensional equations appropriate to laminar and turbulent boundary layers for boundary conditions with an external flow which fluctuates in magnitude; it can readily be extended to the more general case. It is based on the numerical solution of the governing boundary-layer equations by an efficient two-point finite-difference method developed by Keller and Cebeci [1]. An eddy-viscosity formulation is used to model the Reynolds shear-stress term. Here, after briefly describing the main features of the method, we provide instructions for the computer program with a listing, and present sample calculations to demonstrate its usage and capabilities for laminar and turbulent unsteady boundary layers with an external flow which fluctuates in magnitude. For further details of the method, the reader is referred to reference 2.</p> <p style="text-align: right;">ORIGINAL PAGE IS OF POOR QUALITY</p>			
17 Key Words (Suggested by Author(s)) Unsteady boundary layer Laminar and turbulent boundary layer Boundary-layer computation		18 Distribution Statement Unlimited STAR Category - 34	
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